Local density of states in disordered two-dimensional electron gases at high magnetic field

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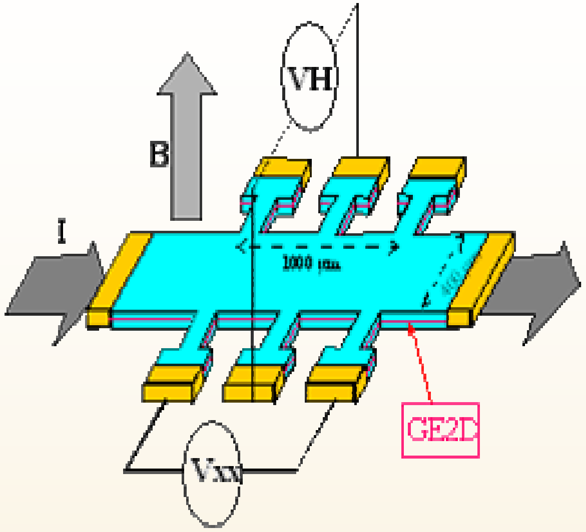
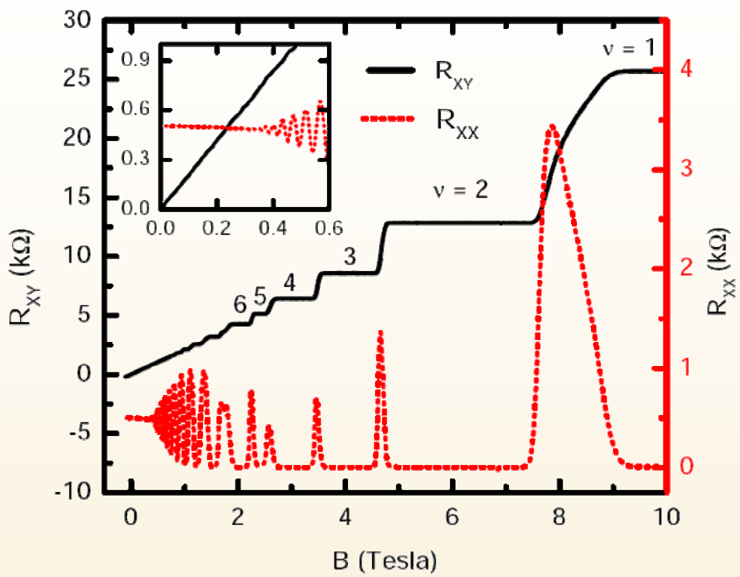
WHY STUDY 2D ELECTRON GASES UNDER MAGNETIC FIELDS NOW?



IQHE :
(1980)

T < 1 Kelvin
B > 1 Tesla



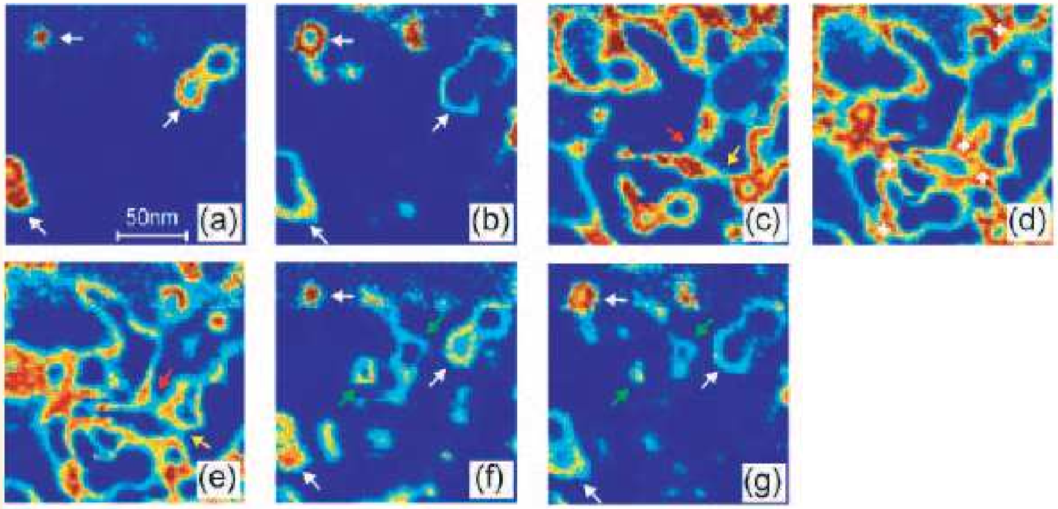
Experiments:

- New effects: microwave induced zero-resistance states
- New probes: local sensing techniques in the IQHE regime
- New systems: graphene, 2D edge states?

WHY STUDY 2D ELECTRON GASES UNDER MAGNETIC FIELDS NOW?

Experiment: Local DOS in the IQHE regime

Hashimoto *et al.*, PRL (2008)



• Percolation features

• Broad structures close to saddle points of the potential landscape

Disorder plays an important role in IQHE!

Theory:

- Many fundamental aspects (e.g. for the IQHE) well understood
- But: how do we calculate stuff? (no quantitative microscopic theory yet!)

→ This talk

Goal:

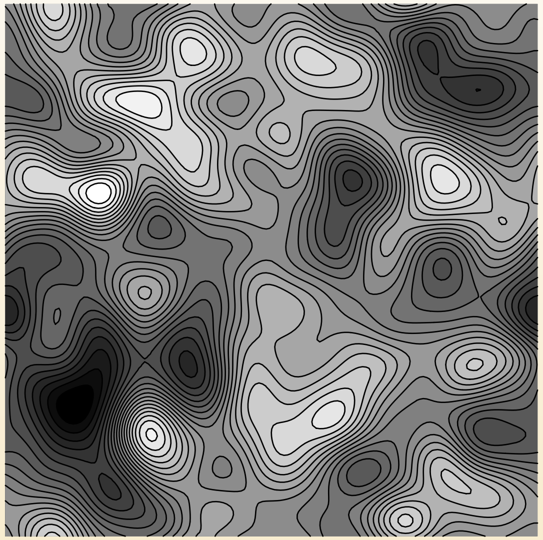
Find an (approximate) analytical solution to the problem

$$H = H_0 + V(\mathbf{r})$$

$$H_0 = \frac{1}{2m^*}\left(-i\hbar\nabla_{\mathbf{r}} - \frac{e}{c}\mathbf{A}(\mathbf{r})\right)^2$$

$$\nabla \times \mathbf{A}(\mathbf{r}) = B\hat{\mathbf{z}}$$

arbitrary potential energy



SUMMARY

- **<u>Basic elements on 2D electron gases under fields</u>**
  - Semiclassical picture
  - Landau levels and wave functions

- **<u>The high field expansion (1st level)</u>**
  - Vortex states and Green's functions
  - Local equilibrium properties

- **<u>The high field expansion (2nd level)</u>**
  - Resummation
  - Local density of states in an arbitrary quadratic potential

- **<u>Outlook:</u>** conclusion and perspectives

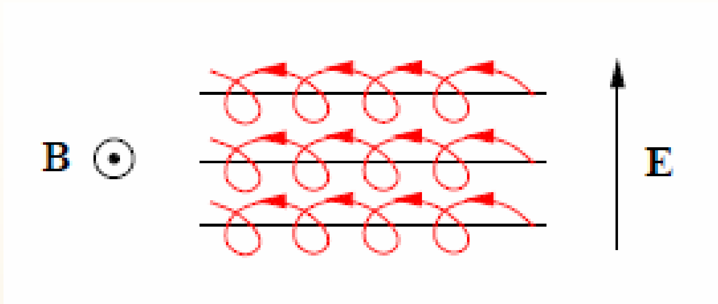
Basics: Landau levels

CLASSICAL MOTION IN HIGH PERPENDICULAR MAGNETIC FIELD

Two degrees of freedom with very different timescales

$\dot{\theta} = \omega_c = |e|B/(m^* c)$ fast cyclotron motion

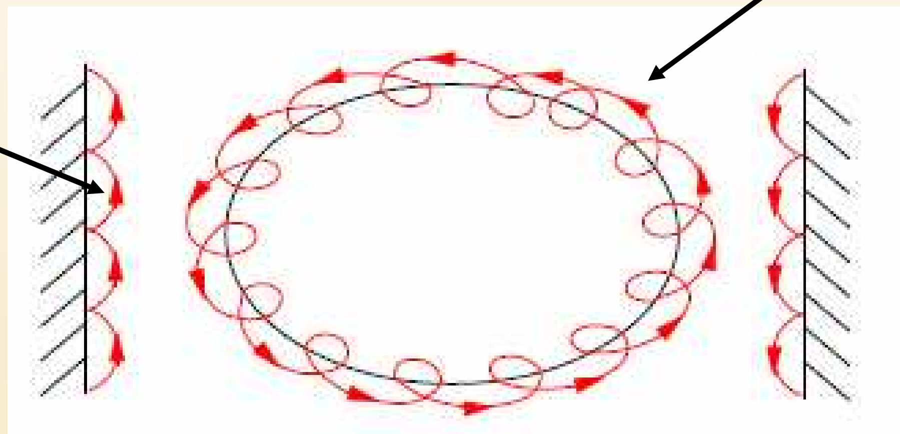
$\mathbf{v}_d = \frac{1}{B}\mathbf{E} \times \hat{\mathbf{z}}$ slow drift



- Decoupling in the limit $B \to \infty$

Disordered bulk: localization on closed equipotential lines

Sharp edges:
delocalized skipping orbits

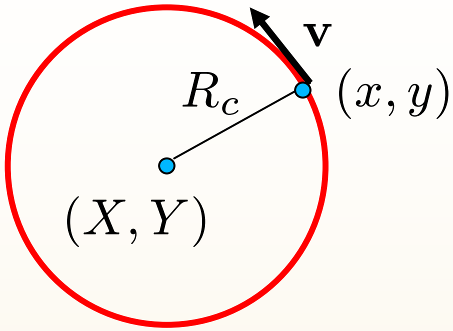


Remark: motion regular and integrable in the limit $B \to \infty$!

Averaging over disordered potential configurations questionable here

SEMICLASSICAL MOTION : THE GUIDING CENTER PICTURE

$$\begin{cases} x = X + \zeta = X + v_y/\omega_c \\ y = Y + \eta = Y - v_x/\omega_c \end{cases}$$



change of variables: $(x, p_x), (y, p_y) \rightarrow (X, Y), (\zeta, \eta)$

$H = \frac{1}{2} m^* \mathbf{v}^2 + V(X + \zeta, Y + \eta)$ then quantization $\begin{cases} [\hat{v}_x, \hat{v}_y] = -i\hbar\omega_c/m^* \\ [\hat{X}, \hat{Y}] = i l_B^2 \end{cases}$

Semiclassical high field picture (V smooth) $l_B^2 = \hbar c/(|e|B) \rightarrow 0$

$[\hat{X}, \hat{Y}] \rightarrow 0$ *X* and *Y* (center coordinates) treated as classical variables:

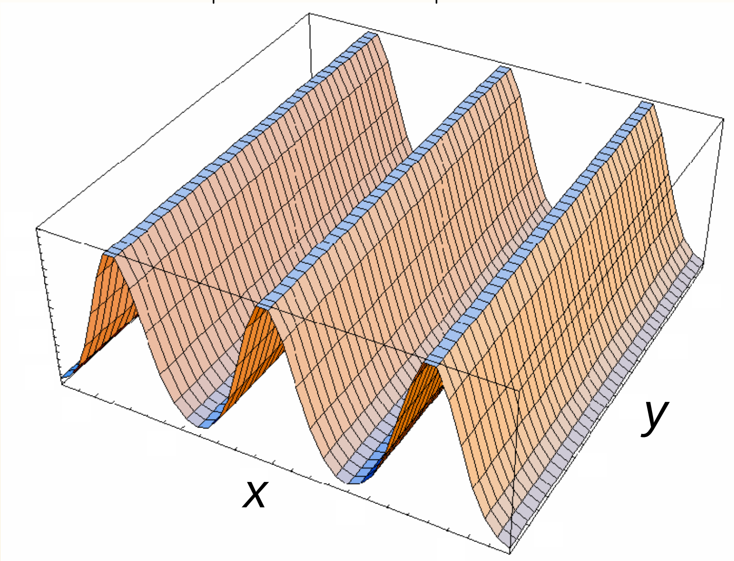
Effective energy: $E_{n,\mathbf{R}} = \hbar\omega_c(n + 1/2) + V(\mathbf{R})$

Limitations:
- No quantization of energies for a closed system (e.g. quantum dot)
- No dissipation of an open system (e.g. no tunneling in QPC)
- Problems to formulate a consistent transport theory
- Captures only the high temperature regime

SCHRÖDINGER EQUATION IN A MAGNETIC FIELD: LANDAU STATES

$$H_0 = \frac{1}{2m^*}\left(-i\hbar\nabla_{\mathbf{r}} - \frac{e}{c}\mathbf{A}(\mathbf{r})\right)^2 \quad \text{with} \quad \nabla \times \mathbf{A}(\mathbf{r}) = B\hat{\mathbf{z}}$$

$\left|\Psi_{n,k}(x,y)\right|^2$



Landau states: Landau (1930)

$$E_{n,k} = \hbar\omega_c(n + \tfrac{1}{2})$$

$$\Psi_{n,k}(x,y) = e^{iky} \exp\left[-\frac{(x-kl_B^2)^2}{2l_B^2}\right] H_n\left(\frac{x-kl_B^2}{l_B}\right)$$

- Translationally invariant along *y*
- Localized on the scale l_B along *x*

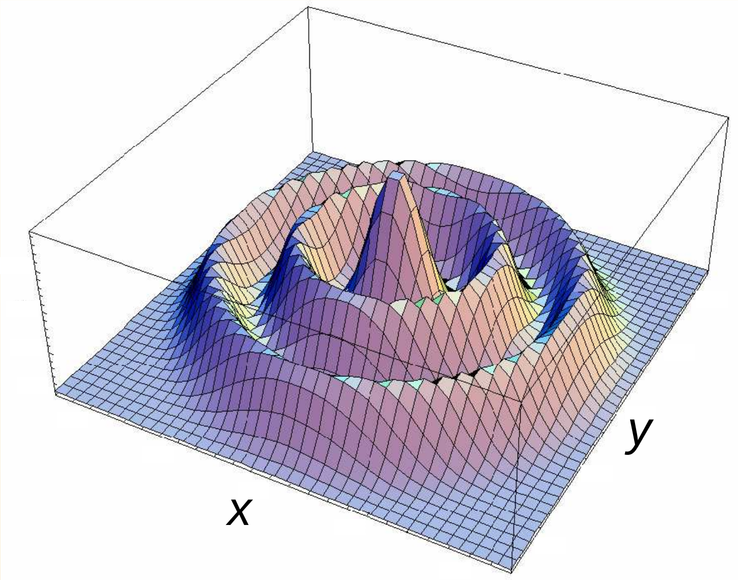
Remarks:

- Huge degeneracy of Landau levels
- Magnetic field enters in wave functions only via $l_B = \sqrt{\hbar c/|e|B}$
- Landau states problematic for quantum/classical correspondence

SCHRÖDINGER EQUATION IN A MAGNETIC FIELD: CIRCULAR STATES

- Other possible eigenstates of H_0

$\left|\Psi_{n,l}(r,\theta)\right|^2$



Circular states:

$$E_{n,l} = \hbar\omega_c\left(n + \frac{|l|-l+1}{2}\right)$$

$$\Psi_{n,l}(r,\theta) = r^{|l|} \exp\left[\frac{-r^2}{4l_B^2}\right] L_n^{|l|}\left(\frac{r^2}{2l_B^2}\right) e^{il\theta}$$

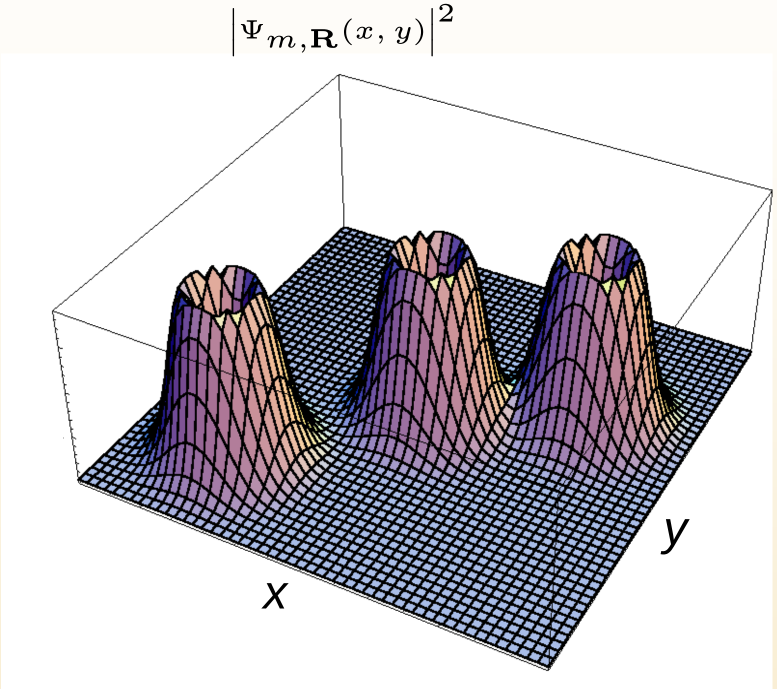
- Rotationally invariant around the origin
- Localized on a scale l_B along r

Remark:

- States still problematic for quantum/classical correspondence at $l_B \to 0$

SCHRÖDINGER EQUATION IN A MAGNETIC FIELD: VORTEX STATES

- Other possible eigenstates of H_0



Vortex states:

$$E_{m,\mathbf{R}} = \hbar\omega_c(m+\tfrac{1}{2})$$

$$\Psi_{m,\mathbf{R}}(\mathbf{r}) = \frac{e^{im\arg(\mathbf{r}-\mathbf{R})}}{\sqrt{2\pi m!}\,l_B}\left|\frac{\mathbf{r}-\mathbf{R}}{\sqrt{2}l_B}\right|^m e^{-\frac{(\mathbf{r}-\mathbf{R})^2-2i(\mathbf{r}\times\mathbf{R})\cdot\hat{\mathbf{z}}}{4l_B^2}}$$

- No symmetry associated with the degeneracy quantum number $\mathbf{R}$
- Localized on a scale l_B around $\mathbf{R}$

Remarks:

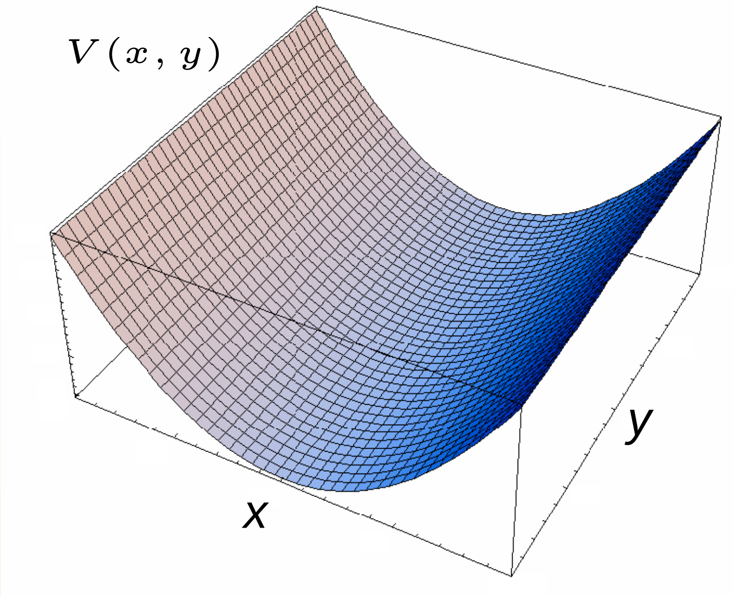
- Nonorthogonal states: $\langle m_1,\mathbf{R_1}|m_2,\mathbf{R_2}\rangle = \delta_{m_1,m_2}\, e^{-\frac{(\mathbf{R}_1-\mathbf{R}_2)^2-2i\hat{\mathbf{z}}\cdot(\mathbf{R}_1\times\mathbf{R}_2)}{4l_B^2}}$
- States OK for quantum/classical correspondence
- States seldomly used in the literature

Basics:

Lifting the Landau levels degeneracy

SOME EXACT RESULTS: 1D PARABOLIC CONFINEMENT

- 1D Parabolic potential $H = H_0 + V(x) = H_0 + \frac{1}{2} m^* \omega_0^2 x^2$



Modified Landau states:

$$E_{n,k} = \hbar\Omega \left(n + \tfrac{1}{2}\right) + V(kL^2)$$

$$\Psi_{nk}(\mathbf{r}) = e^{-iky} \exp\left[-\frac{\left(x - \frac{\omega_c}{\Omega} kL^2\right)^2}{2L^2}\right] H_n\left(\frac{x - \frac{\omega_c}{\Omega} kL^2}{L}\right)$$

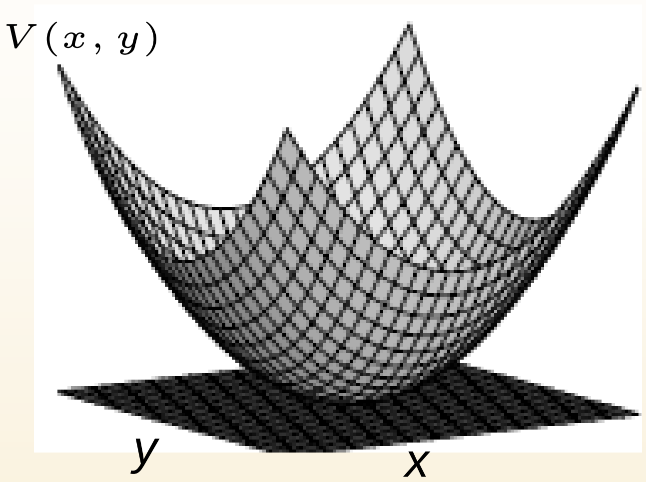
where $\Omega = \sqrt{\omega_c^2 + \omega_0^2}$ and $L = \sqrt{\hbar / m^* \Omega}$

Remarks:

- Degeneracy is fully lifted by $V(x)$
- Eigenfunctions localized around equipotential lines $X = kl_B^2$
- Drift velocity $v_y(X) = \frac{1}{\hbar} \frac{dE_{n,k}}{dk}$

SOME EXACT RESULTS: 2D PARABOLIC CONFINEMENT

- 2D Parabolic potential $H = H_0 + V(r) = H_0 + \frac{1}{2} m^* \omega_0^2 r^2$



Fock (1928) & Darwin (1930)

Fock-Darwin states = modified circular states:

$$E_{n,l} = \hbar\Omega \left(n + \frac{|l|+1}{2} \right) - \frac{l}{2} \hbar\omega_c$$

$$\Psi_{n,l}(r,\theta) = r^{|l|} \exp\left[\frac{-r^2}{4L^2} \right] L_n^{|l|} \left(\frac{r^2}{2L^2} \right) e^{il\theta}$$

where $\Omega = \sqrt{\omega_c^2 + 4\omega_0^2}$ and $L = \sqrt{\hbar / m^* \Omega}$

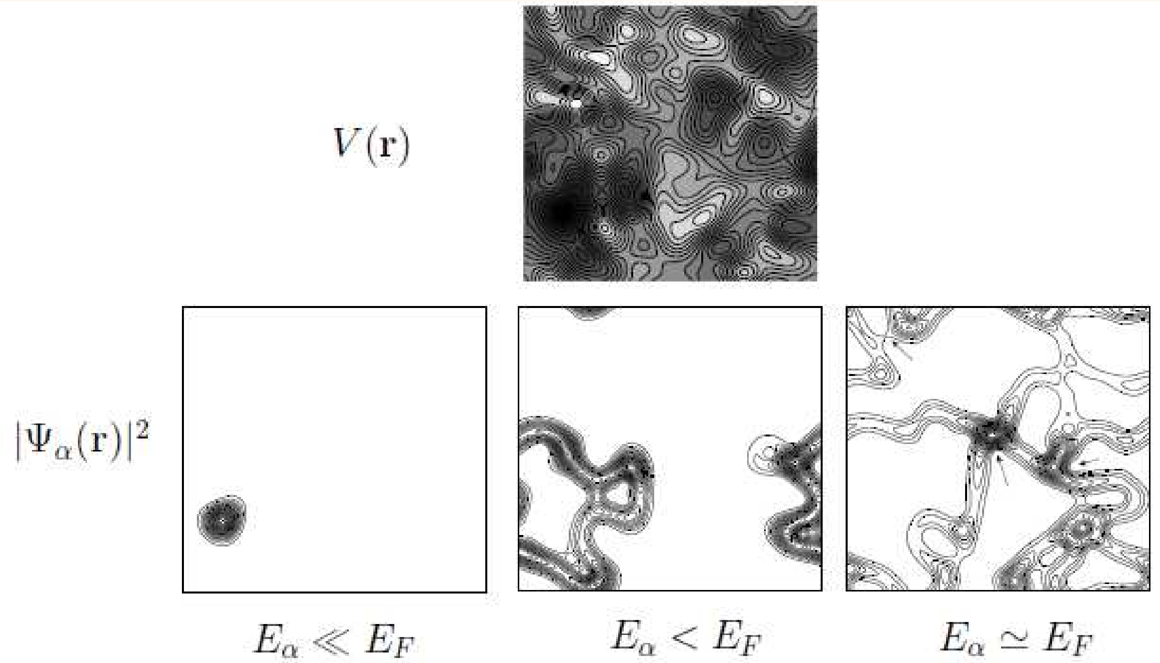
Remarks:

- Energy spectrum entirely discrete
- Degeneracy lifted in general
- Eigenfunctions fully localized

WITH DISORDERED POTENTIAL

- Numerical solution:
  - Confirms the (semiclassical) intuition
  - But not very practical

Kramer *et al.*, Phys. Rep. (2005)



Is there an analytical (approximate) quantum solution at high field ?

The high field expansion (1st level)

How to formulate a quantum theory at finite l_B

Champel, Florens, and Canet, PRB (2008)

MOTIVATION FOR A HIGH MAGNETIC FIELD EXPANSION

At large magnetic field:

- Magnetic length $l_B = 8 \text{ nm at } 10 \text{ T}$
- Correlation length of the disordered potential in heterostructures: $\xi \geq 100 \text{ nm}$

The random potential is smooth on the scale l_B

The idea of using l_B/ξ *as a small parameter is not new. The real challenge is to go beyond the strict limit* $l_B/\xi = 0$ *!*

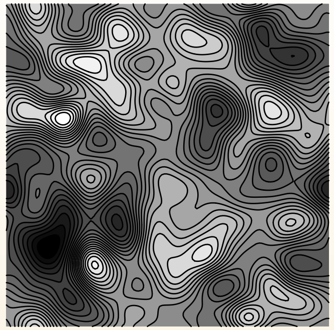
Some attempts:

- *Effective Hamiltonian theory*
  - limited to energy
  - includes only virtual transitions = no Landau-level mixing taken into account

Haldane & Yang, PRL (1997)
Apenko & Lozovik, J. of Phys. C (1984)

SOME CLUE FOR A HIGH MAGNETIC FIELD EXPANSION

$$H = H_0 + V(\mathbf{r})$$ arbitrary potential energy



Modulus of the basis states

Landau basis

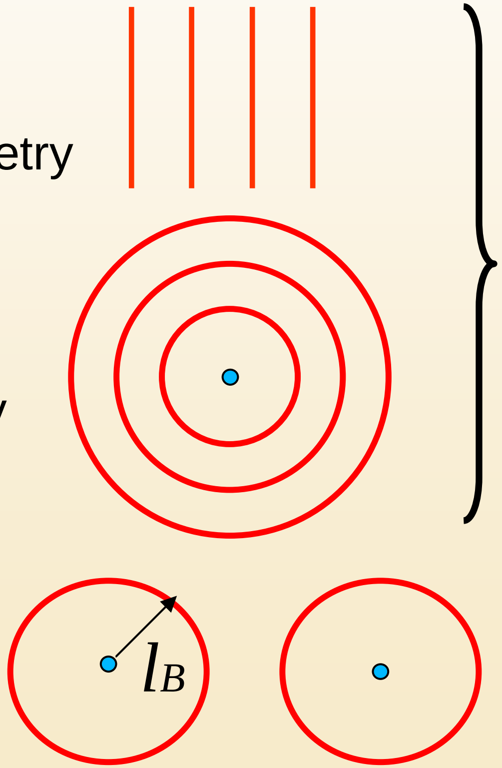
Translation symmetry

Circular basis

Rotation symmetry

Vortex basis

No Symmetry



These states are highly symmetric: conflict with random potential

No particular symmetry for the degeneracy space

⇨ **Better starting point**

Champel & Florens, PRB (2007)

THE HIGH FIELD EXPANSION: METHOD

Vortex representation:

$$\Psi_{m,\mathbf{R}}(\mathbf{r}) = \langle \mathbf{r}|m,\mathbf{R}\rangle$$

Note: overcomplete coherent state basis

$$\sum_{m=0}^{+\infty}\int \frac{d^2\mathbf{R}}{2\pi l_B^2}|m,\mathbf{R}\rangle\langle m,\mathbf{R}| = 1$$

Vortex states can adapt to an arbitrary shape of $V(\mathbf{r})$

Mathematically:

Champel & Florens, PRB (2007)

- Matrix elements of $V(\mathbf{r})$ can be written in a l_B/ξ expansion:

$$\langle \mathbf{R_1}, m_1|V|\mathbf{R_2}, m_2\rangle = \langle \mathbf{R_1}|\mathbf{R_2}\rangle \overbrace{\sum_{n=0}^{+\infty}\left(\frac{l_B}{\sqrt{2}}\right)^n v^{(n)}_{m_1;m_2}(\mathbf{R_{12}})}^{v_{m_1;m_2}(\mathbf{R}_{12})}$$

where

$$\begin{cases} v^{(0)}_{m_1;m_2}(\mathbf{R}) = V(\mathbf{R})\,\delta_{m_1,m_2}, \quad v^{(1)}_{m_1,m_2}(\mathbf{R}) \sim |\nabla V(\mathbf{R})|\,\delta_{m_1,m_2\pm 1} \;\cdots \\ \mathbf{R}_{12} = [\mathbf{R}_1 + \mathbf{R}_2 + i\,(\mathbf{R}_2 - \mathbf{R}_1)\times \hat{\mathbf{z}}]\,/2 \end{cases}$$

VORTEX GREEN'S FUNCTIONS AND DYSON EQUATION

Define:

$$\langle m_1, \mathbf{R_1} | G_0 | m_2, \mathbf{R_2} \rangle = \langle m_1, \mathbf{R_1} | (\omega - \hat{H}_0)^{-1} | m_2, \mathbf{R_2} \rangle = \frac{\delta_{m_1,m_2} \langle \mathbf{R}_1 | \mathbf{R}_2 \rangle}{\omega - E_{m_1} + i0^+}$$

With potential V (Dyson equation):

$$\left(\omega - E_{m_1} + i0^+\right) \langle m_1, \mathbf{R_1} | G | m_2, \mathbf{R_2} \rangle = \langle m_1, \mathbf{R_1} | m_2, \mathbf{R_2} \rangle + \sum_{m_3=0}^{+\infty} \int \frac{d^2\mathbf{R}_3}{2\pi l_B^2} \langle m_1, \mathbf{R_1} | V | m_3, \mathbf{R_3} \rangle \langle m_3, \mathbf{R_3} | G | m_2, \mathbf{R_2} \rangle$$

Champel, Florens, and Canet, PRB (2008)

The solution takes necessarily the form:

$$\langle m_1, \mathbf{R}_1 | G | m_2, \mathbf{R}_2 \rangle = \langle \mathbf{R_1} | \mathbf{R_2} \rangle g_{m_1;m_2}(\mathbf{R}_{12})$$

For coinciding points $\mathbf{R}_1 = \mathbf{R}_2 = \mathbf{R}$

$$\left(\omega - E_{m_1} + i0^+\right) g_{m_1;m_2}(\mathbf{R}) = \delta_{m_1,m_2} + \sum_{m_3=0}^{+\infty} \int \frac{d^2\mathbf{R}_3}{2\pi l_B^2} v_{m_1;m_3}(\mathbf{R}_{13}) g_{m_3;m_2}(\mathbf{R}_{32}) \left|\langle \mathbf{R} | \mathbf{R}_3 \rangle\right|^2$$

DYSON EQUATION IN VORTEX REPRESENTATION : SOLUTION

- Expand in Taylor series and integrate to get

Champel *et al.*, PRB (2008)

$$\left(\omega - E_{m_1} + i0^+\right) g_{m_1;m_2}(\mathbf{R}) = \delta_{m_1,m_2} + \sum_{m_3=0}^{+\infty} \sum_{k=0}^{+\infty} \left(\frac{l_B}{\sqrt{2}}\right)^{2k} \frac{1}{k!} \left(\partial_X - i\partial_Y\right)^k v_{m_1;m_3}(\mathbf{R}) \times \left(\partial_X + i\partial_Y\right)^k g_{m_3;m_2}(\mathbf{R})$$

Solution under the form:

$$g_{m_1;m_2}(\mathbf{R}) = \sum_{n=0}^{+\infty} \left(\frac{l_B}{\sqrt{2}}\right)^n g^{(n)}_{m_1;m_2}(\mathbf{R})$$

<u>Lowest order result:</u>
$(n = 0)$

$$g^{(0)}_{m_1;m_2}(\mathbf{R}) = \frac{\delta_{m_1,m_2}}{\omega - E_{m_1} - V(\mathbf{R}) + i0^+}$$

<u>To all orders:</u> closed recursion!

$$g^{(n)}_{m_1;m_2}(\mathbf{R}) = g^{(0)}_{m_1;m_1}(\mathbf{R}) \sum_{l<n,j,k,m_3,p} \frac{\delta_{n,2k+j+l}}{k!} \frac{(m_1+p)!}{\sqrt{m_1!m_3!}} \frac{\delta_{m_1+p,m_3+j-p}}{p!(j-p)!} \times \left(\partial_X - i\partial_Y\right)^{k+j-p} \left(\partial_X + i\partial_Y\right)^p V(\mathbf{R}) \left(\partial_X + i\partial_Y\right)^k g^{(l)}_{m_3;m_2}(\mathbf{R})$$

HOW TO GET PHYSICAL QUANTITIES AT EQUILIBRIUM?

★ Change of representation:

$$G(\mathbf{r},\mathbf{r}',\omega) = \int\frac{d^2\mathbf{R}_1}{2\pi l_B^2}\int\frac{d^2\mathbf{R}_2}{2\pi l_B^2}\sum_{m_1,m_2}\Psi^*_{m_2,\mathbf{R}_2}(\mathbf{r}')\Psi_{m_1,\mathbf{R}_1}(\mathbf{r})g_{m_1;m_2}(\mathbf{R}_{12})\langle\mathbf{R}_1|\mathbf{R}_2\rangle$$

Magic formula: simple (quasilocal) connection of electronic Green's functions to vortex Green's functions

$$G(\mathbf{r},\mathbf{r}',\omega) = \int\frac{d^2\mathbf{R}}{2\pi l_B^2}\sum_{m_1,m_2}\Psi^*_{m_2,\mathbf{R}}(\mathbf{r}')\Psi_{m_1,\mathbf{R}}(\mathbf{r})\sum_{k=0}^{+\infty}\frac{1}{k!}\left(-\frac{l_B^2}{2}\Delta_{\mathbf{R}}\right)^k g_{m_1;m_2}(\mathbf{R})$$

- Local charge density: $$\rho(\mathbf{r}) = -i\int\frac{d\omega}{2\pi}\,G^<(\mathbf{r},\mathbf{r},\omega)$$

- Local current density: $$\mathbf{j}(\mathbf{r}) = \int\frac{d\omega}{2\pi}\left[\frac{e\hbar}{2m^*}(\nabla_{\mathbf{r}'}-\nabla_{\mathbf{r}}) + i\frac{e^2}{m^*c}\mathbf{A}\right]G^<(\mathbf{r},\mathbf{r}',\omega)\Big|_{\mathbf{r}=\mathbf{r}'}$$

where $$G^<(\omega) = n_F(\omega)\left[G^A(\omega)-G^R(\omega)\right]$$

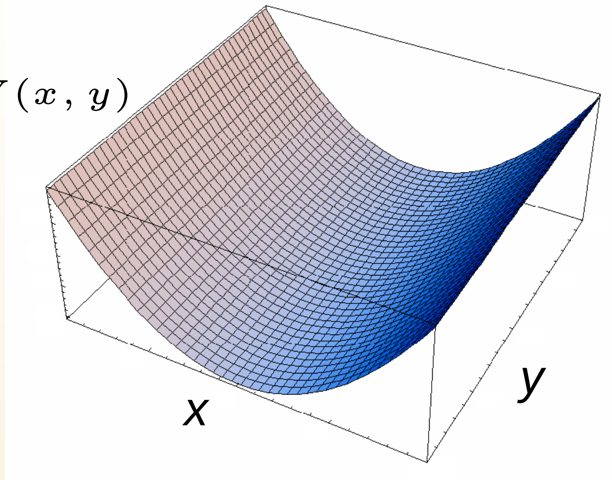
Equilibrium properties

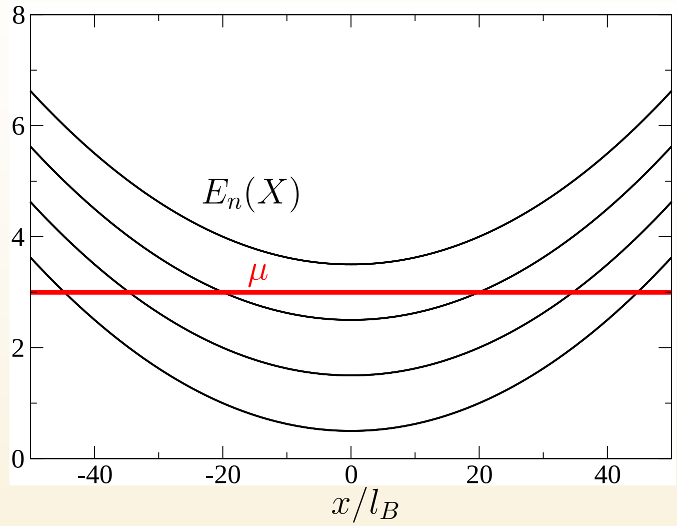
Validation of the theory

Champel, Florens, and Canet, PRB (2008)

COMING BACK TO THE 1D CONFINING POTENTIAL MODEL

- 1D Parabolic potential $H = H_0 + V(x) = H_0 + \frac{1}{2} m^* \omega_0^2 x^2$

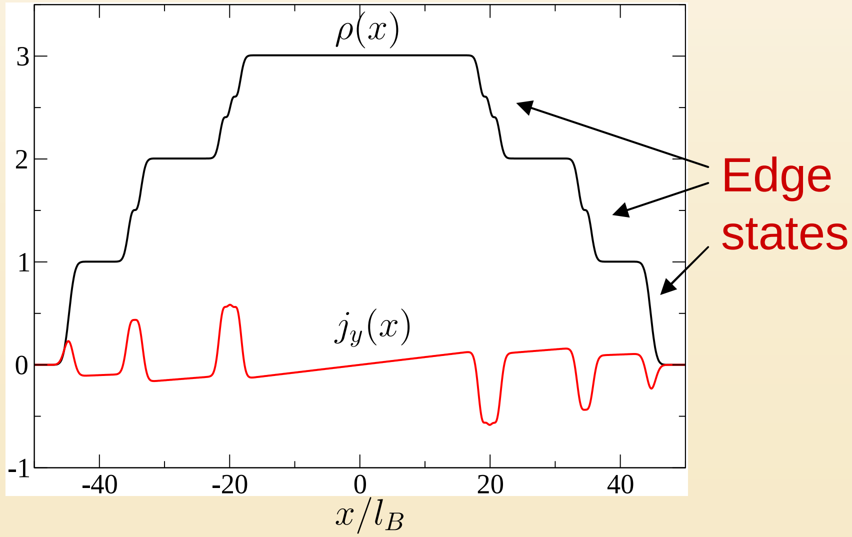




$$E_{n,k} = \hbar\Omega\left(n+\tfrac{1}{2}\right) + V(kL^2)$$

$$\Psi_{nk}(\mathbf{r}) = e^{-iky} \exp\left[-\frac{\left(x-\frac{\omega_c}{\Omega}kL^2\right)^2}{2L^2}\right] H_n\left(\frac{x-\frac{\omega_c}{\Omega}kL^2}{L}\right)$$

$\rho(\mathbf{r})$ and $\mathbf{j}(\mathbf{r})$ known exactly



ELECTRONIC CHARGE DENSITY: THE VORTEX THEORY RESULT

★ Leading order of vortex expansion:

$$\xi_m(\mathbf{R}) = E_m + V(\mathbf{R})$$

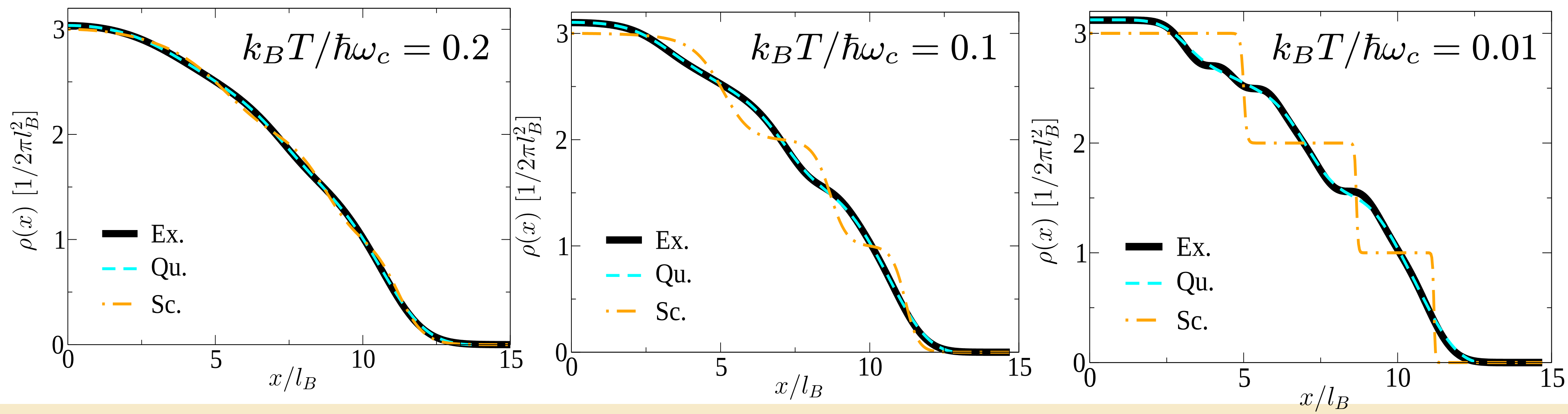
$$\rho_{\text{Qu.}}(\mathbf{r}) = \int \frac{d^2\mathbf{R}}{2\pi l_B^2} \sum_m n_F[\xi_m(\mathbf{R})] |\Psi_{m,\mathbf{R}}(\mathbf{r})|^2$$

NEW

★ Leading order of semiclassical expansion (point-like wave function for $l_B = 0$)

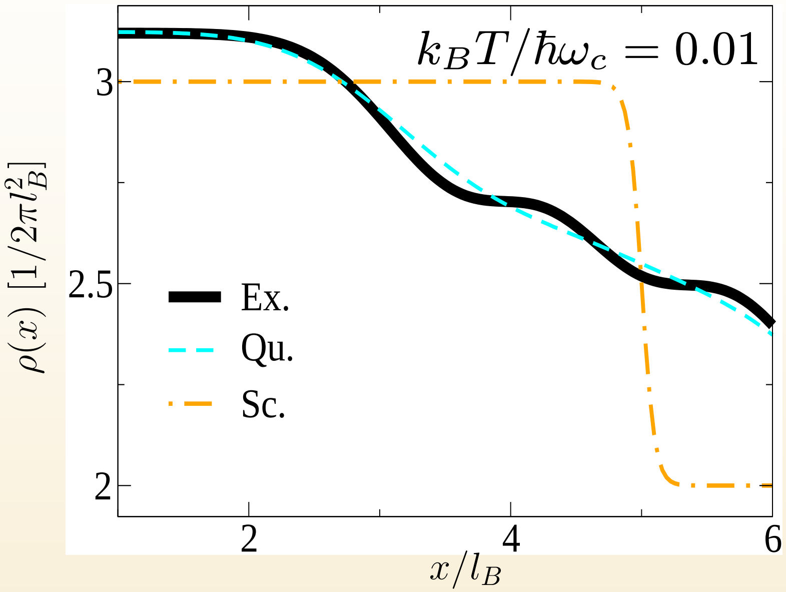
$$\rho_{\text{Sc.}}(\mathbf{r}) = \frac{1}{2\pi l_B^2} \sum_m \left(n_F[\xi_m(\mathbf{r})] + O(l_B^2) \right)$$

► Comparing with exact expression of 1D confinement model:



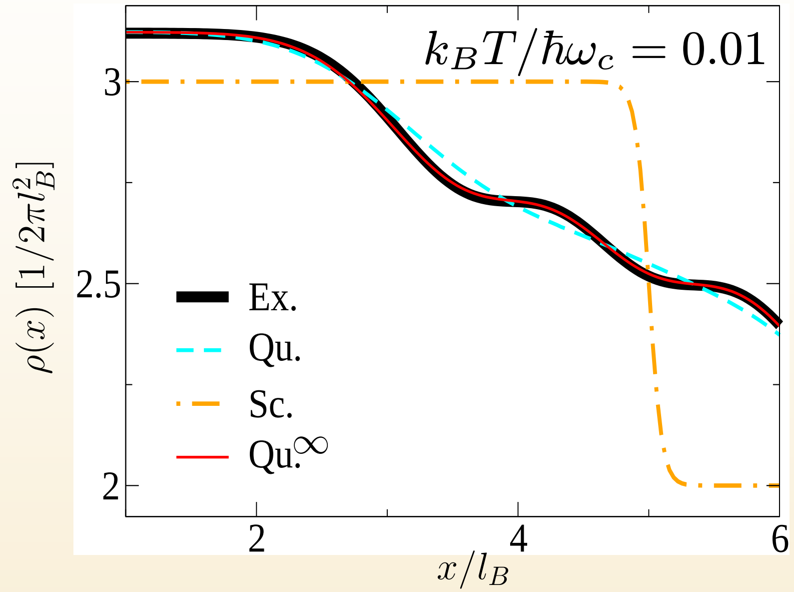
ELECTRONIC CHARGE DENSITY: RECOVERING THE FULL QUANTUM RESULT

Zooming in: deviations from terms like $\left(l_B^2 \Delta_{\mathbf{r}}\right)^k \rho_{\text{Qu.}}(\mathbf{r}) = O(1)$



ELECTRONIC CHARGE DENSITY: RECOVERING THE FULL QUANTUM RESULT

Zooming in: deviations from terms like $\left(l_B^2 \Delta_{\mathbf{r}}\right)^k \rho_{\text{Qu.}}(\mathbf{r}) = O(1)$



- Infinite order resummation of a class of terms: it can be done and it works!

$$\rho_{\text{Qu.}\infty}(\mathbf{r}) = \int \frac{d^2\mathbf{R}}{2\pi l_B^2} \sum_{m=0}^{+\infty} \frac{n_F[\xi_m(\mathbf{R})]}{\pi m! l_B^2} A_m(\mathbf{R}-\mathbf{r}) \exp\left[-\frac{(\mathbf{R}-\mathbf{r})^2}{l_B^2}\right]$$

where $A_m(\mathbf{R}) = \frac{\partial^m}{\partial s^m}\left(\frac{1}{1+s}\exp\left[\frac{\mathbf{R}^2}{l_B^2}\frac{2s}{1+s}\right]\right)_{s=0}$: special polynomial

The high field expansion (2nd level)

Resummation and curvature effects

Champel & Florens, PRB (2009)

Champel & Florens, ArXiv:0904.3262 (unpublished)

MODIFIED VORTEX GREEN'S FUNCTION

Change in functions

$$\begin{cases} \tilde{g}_{m_1;m_2}(\mathbf{R}) = e^{-\frac{l_B^2}{4}\Delta_{\mathbf{R}}} g_{m_1;m_2}(\mathbf{R}) \\ \tilde{v}_{m_1;m_2}(\mathbf{R}) = e^{-\frac{l_B^2}{4}\Delta_{\mathbf{R}}} v_{m_1;m_2}(\mathbf{R}) \end{cases}$$

$$\Rightarrow \quad G(\mathbf{r},\mathbf{r}',\omega) = \int \frac{d^2\mathbf{R}}{2\pi l_B^2} \sum_{m_1,m_2} \tilde{g}_{m_1;m_2}(\mathbf{R})\, e^{-\frac{l_B^2}{4}\Delta_{\mathbf{R}}} \left[\Psi^*_{m_2,\mathbf{R}}(\mathbf{r}')\Psi_{m_1,\mathbf{R}}(\mathbf{r})\right]$$

Modified Dyson's equation (forgetting Landau level mixing for simplicity)

$$(\omega - E_m + i0^+)\tilde{g}_m(\mathbf{R}) = 1 + e^{i\frac{l_B^2}{2}(\partial_X^v \partial_Y^g - \partial_Y^v \partial_X^g)}\tilde{v}_m(\mathbf{R})\tilde{g}_m(\mathbf{R})$$

- Trivial for 1D potentials: $\tilde{g}_m(\mathbf{R}) = [\omega - E_m - \tilde{v}_m(\mathbf{R}) + i0^+]^{-1}$

resummation of terms accounted for by change in functions!

★ Rigorous quantum formulation of an early idea by Trugman, PRB (1983)

INCLUDING CURVATURE EFFECTS

Some non trivial questions:

- How to get quantized energies for a closed system?
- How to get dissipation of an open system (QPC)?

Answer: everything is encoded in quadratic terms of the potential V!

$$V(\mathbf{R}) = V(\mathbf{R}_0) + [\mathbf{R} - \mathbf{R}_0] \cdot \nabla V(\mathbf{R}_0) + \tfrac{1}{2} \left[(\mathbf{R} - \mathbf{R}_0) \cdot \nabla\right]^2 V(\mathbf{R}_0)$$

Dyson's equation up to second-order derivatives of V:

$$1 = \left[\omega - E_m - V(\mathbf{R}) - \frac{2m+1}{4} l_B^2 \Delta_{\mathbf{R}} V + i0^+\right] \tilde{g}_m(\mathbf{R})$$
$$+ \frac{l_B^4}{8} \left[\partial_Y^2 V \partial_X^2 + \partial_X^2 V \partial_Y^2 - 2\partial_X \partial_Y V \partial_X \partial_Y\right] \tilde{g}_m(\mathbf{R})$$

This ugly equation can be exactly solved! Champel & Florens, PRB (2009)

EXACT SOLUTION FOR ANY QUADRATIC POTENTIAL

Solution (m=0):

$$\tilde{g}_m(\mathbf{R}) = -i\int_0^{+\infty} dt\, \frac{e^{i\frac{\eta(\mathbf{R})}{\gamma}[t-\tan(\sqrt{\gamma}t)/\sqrt{\gamma}]}}{\cos(\sqrt{\gamma}t)} e^{it\left[\omega - V(\mathbf{R}) - l_B^2\Delta V(\mathbf{R})/4 + i0^+\right]}$$

where

$$\gamma = \frac{l_B^4}{4}\left[\partial_{XX}V\partial_{YY}V - (\partial_{XY}V)^2\right]$$

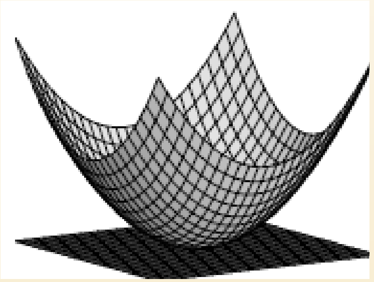
Related to the Gaussian curvature of V

$$\eta(\mathbf{R}) = \frac{l_B^4}{8}\left[\partial_{XX}V(\partial_Y V)^2 + \partial_{YY}V(\partial_X V)^2 - 2\partial_{XY}V\partial_X V\partial_Y V\right]$$

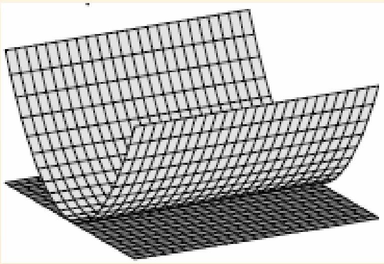
Solution embraces all possible cases of quadratic potentials

⬇

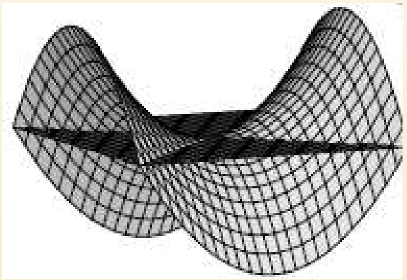
Stability of vortex quantum numbers



$\gamma > 0$



$\gamma = 0$



$\gamma < 0$

Solution periodical in time
➡ energy quantization

Solution with lifetime
➡ tunneling and dissipation

Open and closed quantum mechanics unified!

Champel & Florens, PRB (2009)

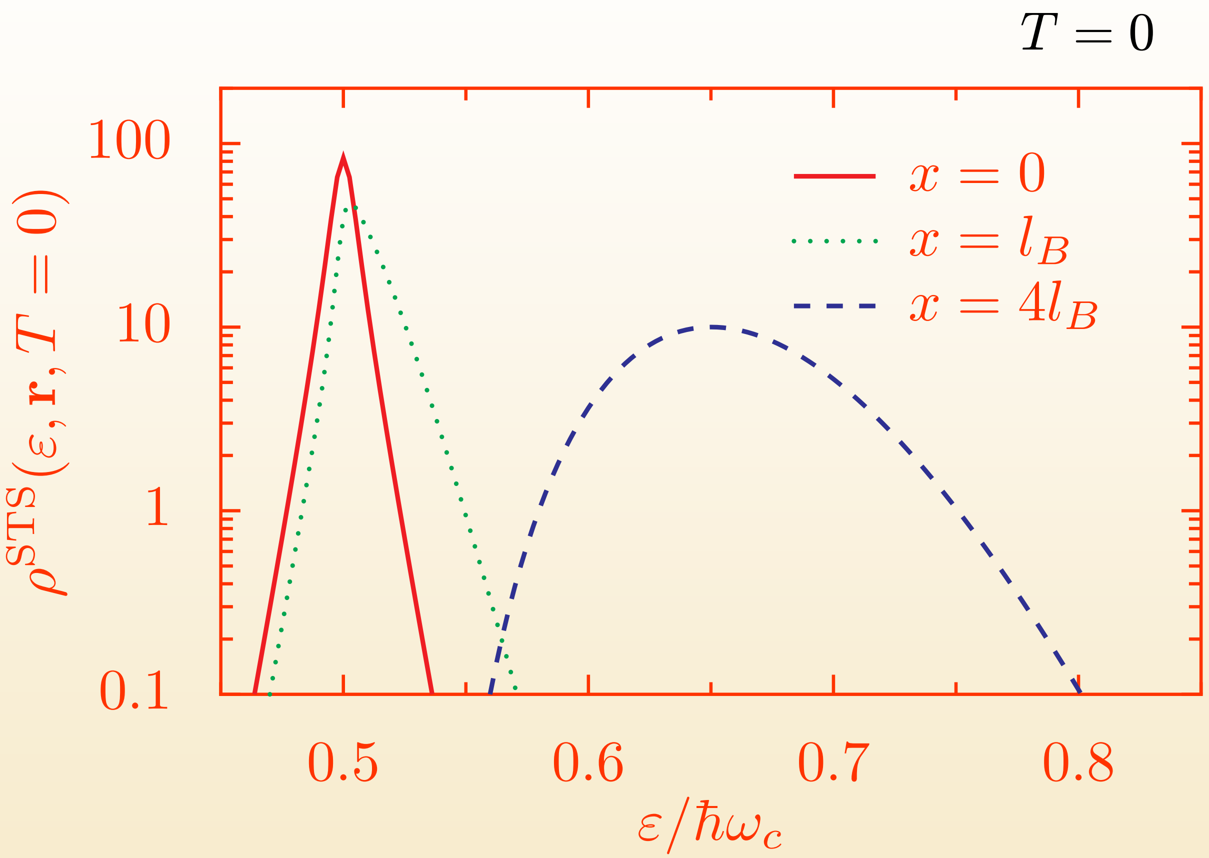
Experimental implications

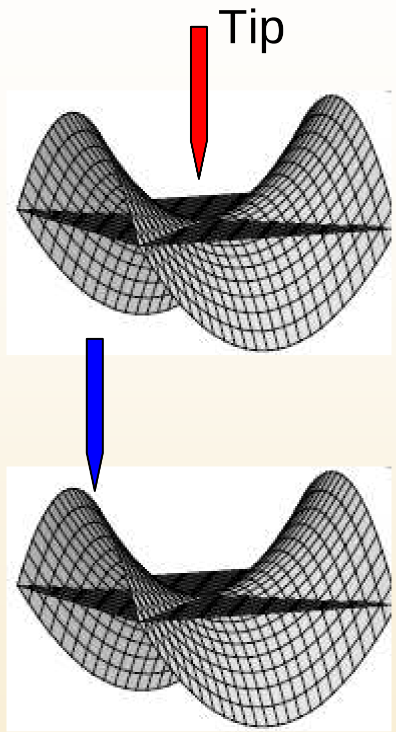
Scanning tunneling spectroscopy

Champel & Florens, ArXiv:0904.3262 (unpublished)

LOCAL DENSITY OF STATES FOR THE SADDLE-POINT POTENTIAL

Model: $V = \frac{1}{2} m^* \omega_0^2 XY$





Quite different shapes (tails and spectral asymmetries) depending on tip position

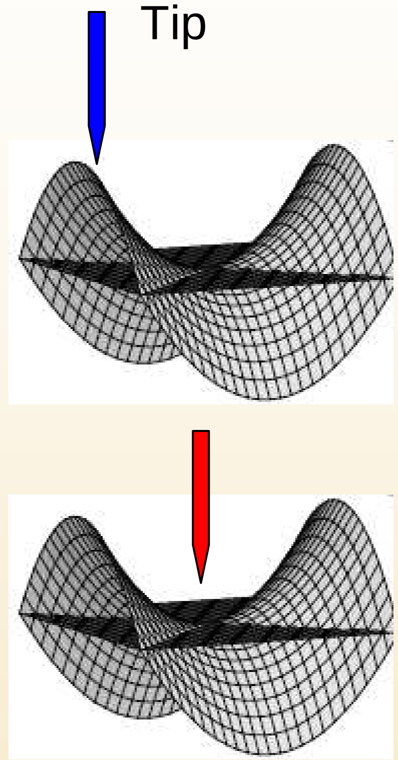
Champel & Florens, ArXiv:0904.3262 (unpublished)

INTERPRETATION: TWO DIFFERENT ENERGY SCALES AT T=0

Champel & Florens, ArXiv:0904.3262 (unpublished)

Two energy scales:

$$\omega_{\rm drift} = l_B |\nabla_{\mathbf{r}} V(\mathbf{r})| \quad \text{vs} \quad \omega_{\rm curv} = 2\sqrt{-\gamma}$$



Various regimes:

- Far from the critical point: **drift dominated**

$$\rho^{STS} \sim \frac{1}{\omega_{\rm drift}} \exp\left[-\left(\frac{\epsilon - V(\mathbf{r})}{\omega_{\rm drift}}\right)^2\right]$$

- Close to the critical point: **curvature dominated**

$$\rho^{STS} \sim \frac{1}{\omega_{\rm curv}} \operatorname{sech}\left[\pi\left(\frac{\epsilon - V(\mathbf{r})}{\omega_{\rm curv}}\right)\right]$$

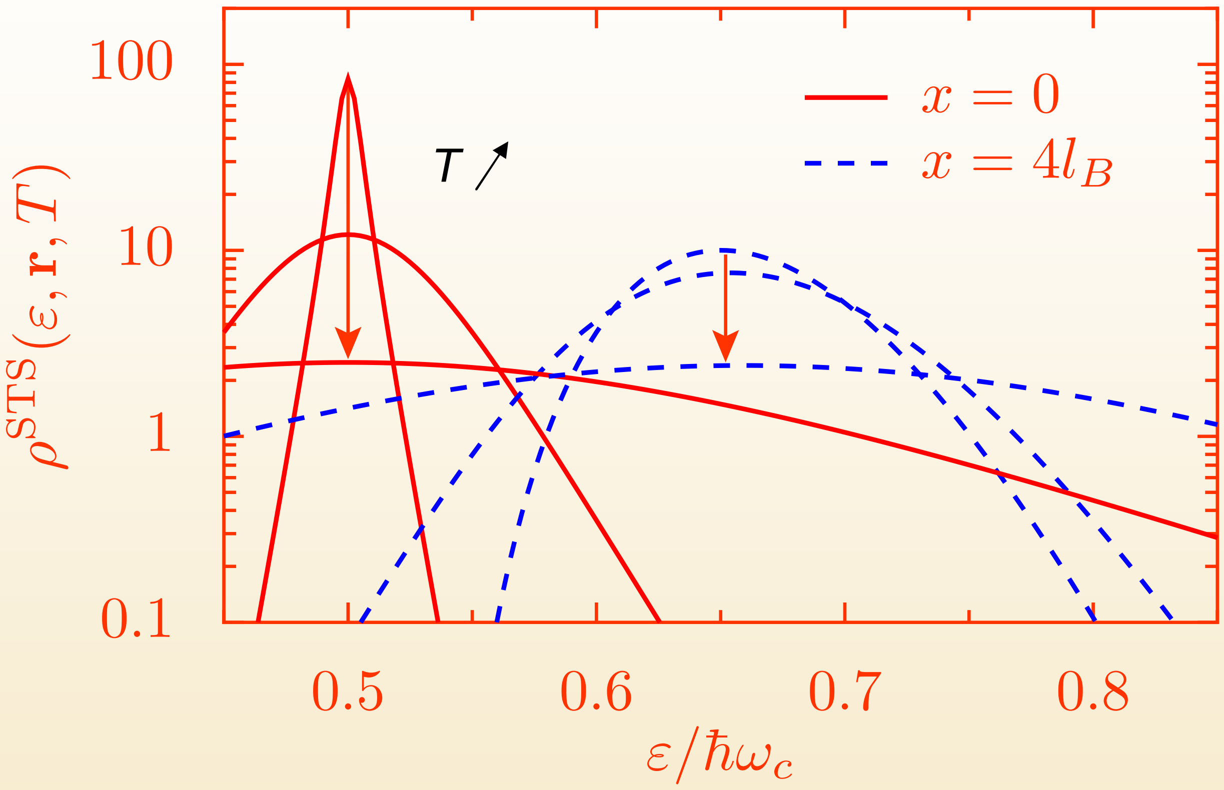
- **Thermal dominated**

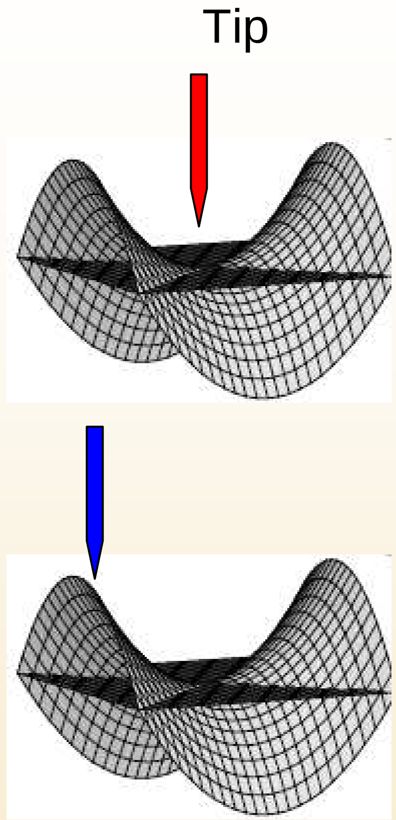
$$\rho^{STS} \sim \frac{1}{4T} \operatorname{sech}^2\left[\pi\left(\frac{\epsilon - V(\mathbf{r})}{2T}\right)\right]$$

New characteristic features

LOCAL DENSITY OF STATES FOR THE SADDLE-POINT POTENTIAL

Thermal effects:





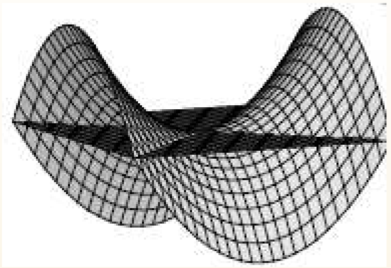
Thermal smearing is more effective close to critical points of V

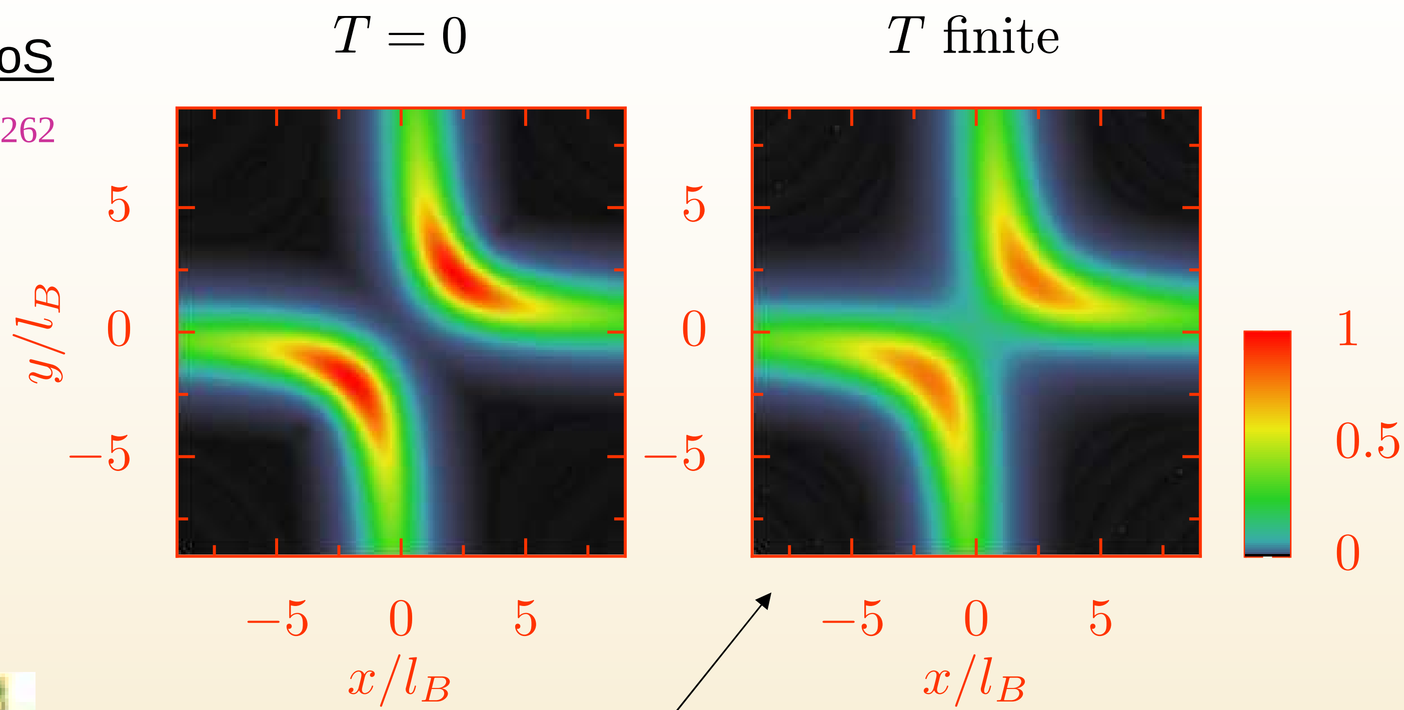
Champel & Florens, ArXiv:0904.3262 (unpublished)

LOCAL DENSITY OF STATES FOR THE SADDLE-POINT POTENTIAL: THERMAL EFFECTS

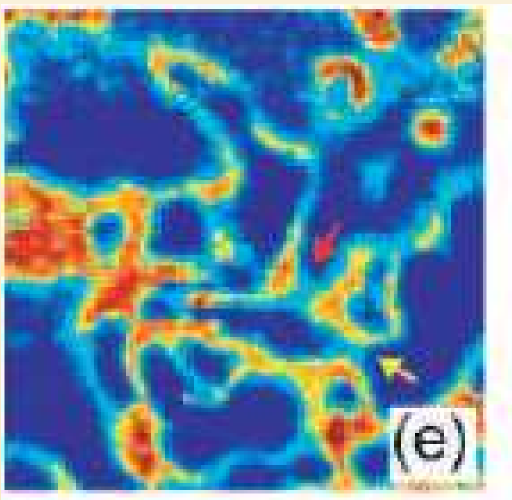
Intensity plots of the LDoS

Champel & Florens, ArXiv:0904.3262





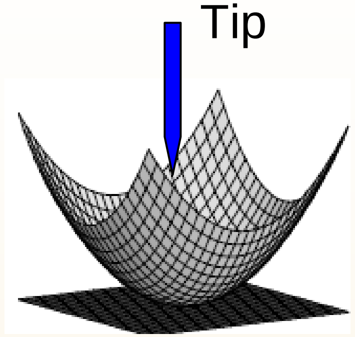
Hashimoto *et al.*, PRL (2008)



Overbroadening of intensity at saddle points

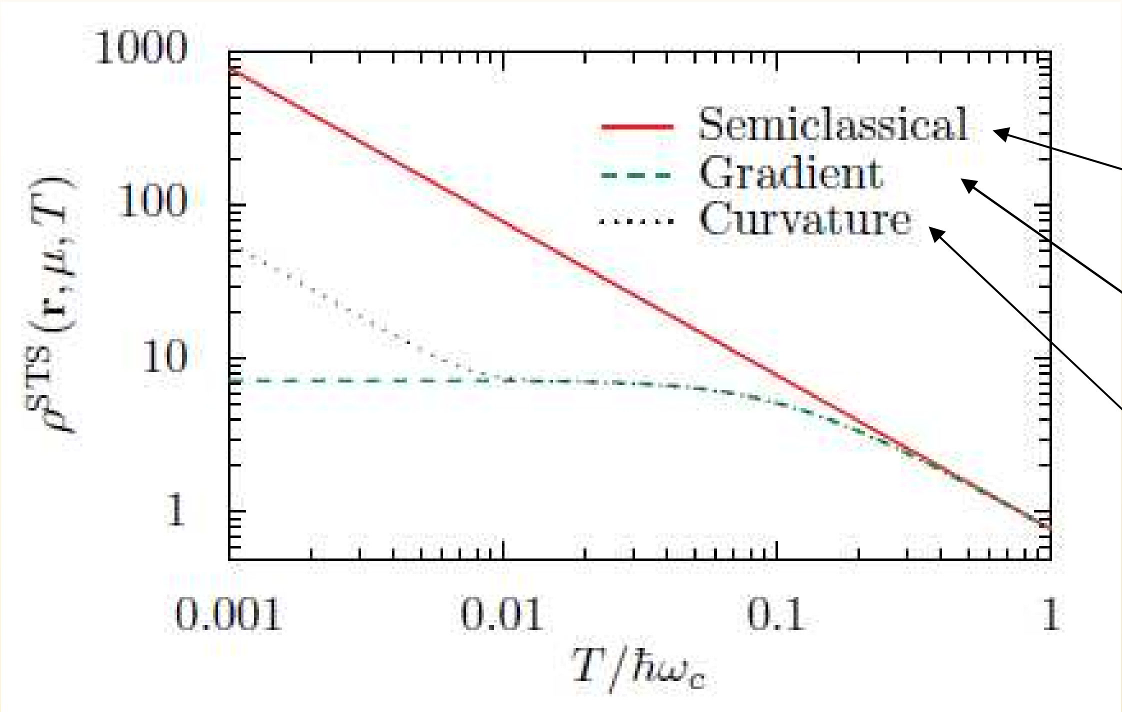
True quantum tunneling states are hard to see experimentally

VALIDITY OF THE RESULTS FOR A DISORDERED POTENTIAL (BEYOND QUADRATIC ORDER)



Example: $V = \frac{1}{2} m^* \omega_0^2 r^2$

LDoS at the center within different approximation schemes



$$\rho(\mathbf{r},\mu,T) = -\frac{1}{2\pi l_B^2} \sum_{m=0}^{+\infty} n_F'[E_m + V(\mathbf{r})]$$

$\gamma = \eta = 0$

exact expression

Existence of a hierarchy of local energy scales → **controlled theory**

Champel & Florens, PRB (2009)

CONCLUSION

- Vortex wave functions are the naturally selected quantum states in high field

- The mathematical foundation of the l_B expansion was established in terms of vortex Green's functions

  - generates trivially the semiclassical expansion
  - provides a fully quantum approach to guiding center ideas
  - unifies closed and open systems (quantization vs dissipation)

- Local equilibrium observables can be calculated accurately from simple and controlled density functionals

  - example of the LDoS expressed in terms of geometric invariants

- *Perspectives:*

  - controlled theory for quantum transport in a disordered potential landscape
  - Graphene