Dynamics of Entanglement

Marcelo França Santos

UFMG – Brasil

(On “lease” to) CQT – Singapore

(and also to) Institut Néel - France

Introduction

- Two words about entanglement
- A couple more about quantum open systems
- Disentanglement dynamics – two-level systems
- Monitoring the reservoir = acquiring information
- No-jump trajectory protects entanglement (a little bit)
- Can we do better than this?
- Locally protecting two qubit entanglement through reservoir monitoring plus something else…
- Longer distance quantum communication
- Experimental candidates

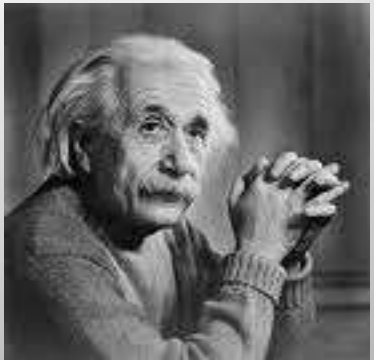
Entanglement

"When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own. **I would not call that one but rather the characteristic trait of quantum mechanics**, the one that enforces its entire departure from classical lines of thought. By the interaction the two representatives [the quantum states] have become **entangled**."

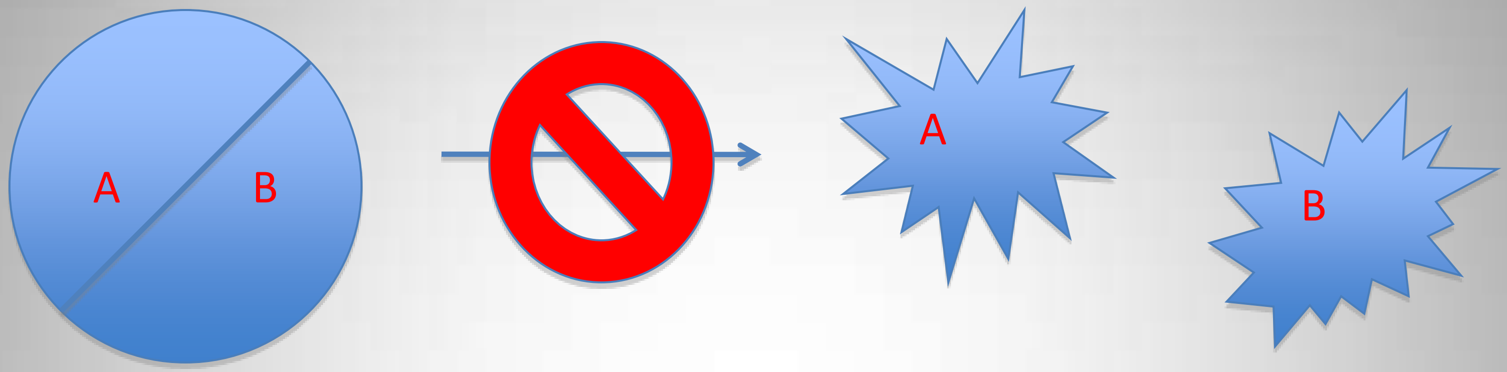
- Schrödinger (1935, Cambridge Philosophical Society)

"Spooky action a distance."

- Einstein (EPR, Phs. Rev., 1935)



Entanglement



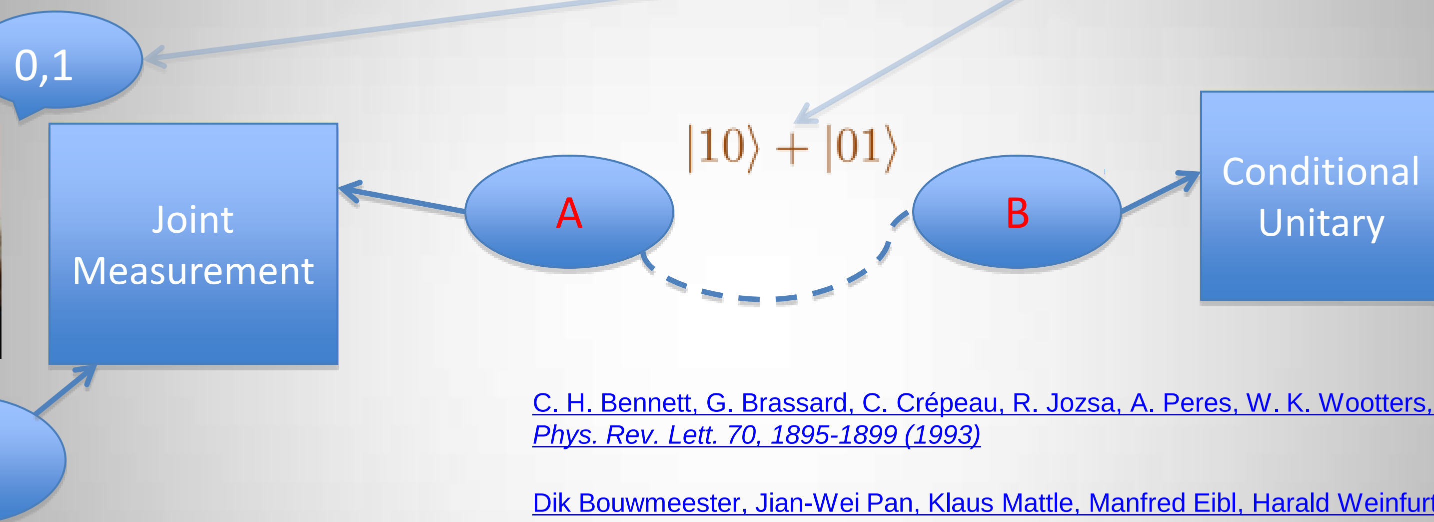
$$|\Psi_{AB}\rangle \neq |\Psi_A\rangle \otimes |\Psi_B\rangle$$

Two or more particles, two or more degrees of freedom, two or more propagation modes (even with a single particle!), etc…

Entanglement as a resource

What is it good for?

Quantum teleportation: from infinite bits to 2 bits + 1 ebit



C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, W. K. Wootters, *Phys. Rev. Lett. 70, 1895-1899 (1993)*

Dik Bouwmeester, Jian-Wei Pan, Klaus Mattle, Manfred Eibl, Harald Weinfurter & Anton Zeilinger
Experimental Quantum Teleportation, Nature 390 (1997) 575-579

$$|\Psi\rangle = \cos\theta|0\rangle + \sin\theta e^{i\phi}|1\rangle$$

Real numbers = infinite bits






Quantum Key Distribution – protected by entanglement

Artur Ekert, Phys. Rev. Lett. 67, 661–663 (1991)

idQuantique, MagicQ, Quintessencelabs

Super dense coding – 2 bits in one qubit

C. Bennett and S.J. Wiesner, Phys. Rev. Lett., 69:2881, 1992

Quantum Computing (?)

Entropy x entanglement (classical x quantum information)

- In classical information theory, entropy is the boss!

For example:

$$H(invested) + H(spent) - H(invested, spent)$$

Entropy x entanglement (classical x quantum information)

- Quantum information theory seems to be more anarchic!

Von-Neumman entropy – does it tell it all (or at all) about entanglement?

$$S(\rho) = -\mathrm{Tr}(\rho \log \rho)$$

For bipartite pure states: yes! (is there hope?)

$$S(\rho_{AB}) = 0 \qquad S(\rho_A) = E(\rho_{AB}) = S(\rho_B)$$

$$|\Psi\rangle = |0\rangle \otimes |0\rangle \qquad S(\rho_A) = 0 = S(\rho_B)$$ Separable!

$$|\Psi\rangle = \frac{|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle}{\sqrt{2}} \qquad S(\rho_A) = 1 = S(\rho_B)$$ Entangled!

If the global state is pure, any local classical indeterminacy (mixture) has to come from entanglement!

For anything else: not really! (no universal measure of entanglement)

Simple example:

$$\rho_{AB} = \frac{1}{2}|00\rangle\langle 00| + \frac{1}{2}|11\rangle\langle 11|$$

$$S(\rho_{AB}) = 1$$

$$\rho_A = \rho_B = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$$

$$S(\rho_A) = -\frac{1}{2}\log\frac{1}{2} - \frac{1}{2}\log\frac{1}{2}$$

$$S(\rho_A) = S(\rho_B) = 1$$

$$|\Psi_{AB}\rangle = \frac{1}{3}|00\rangle + \frac{2\sqrt{2}}{3}|11\rangle$$

$$S(\rho_{AB}) = 0$$

$$\rho_A = \rho_B = \frac{1}{9}|0\rangle\langle 0| + \frac{8}{9}|1\rangle\langle 1|$$

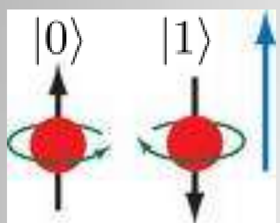
$$S(\rho_A) = -\frac{1}{9}\log\frac{1}{9} - \frac{8}{9}\log\frac{8}{9}$$

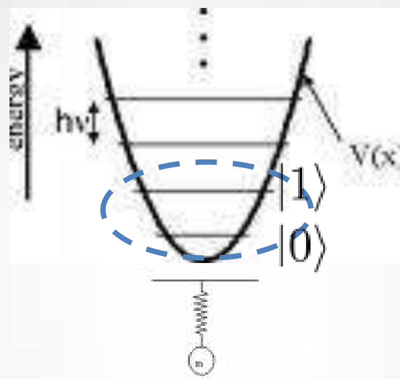
$$S(\rho_A) = 0.5 = S(\rho_B)$$

$$S(\rho_A) + S(\rho_B) - S(\rho_{AB}) = 1$$

However, for 2 x 2 and 2 x 3 there are equivalent measures: concurrence, negativity, entropy of formation, etc…

1 qubit = 1 two level system!





Also:

- Two atomic levels
- One exciton
- One polariton
- Polarization of photons
- Etc

In particular, negativity works fine and is easy to calculate

Idea behind negativity: non-physical transformations that may still produce physical states.

Asher Peres, ***Separability Criterion for Density Matrices, Phys. Rev. Lett. 77, 1413–1415 (1996)***

M. Horodecki, P. Horodecki, R. Horodecki, Separability of Mixed States: Necessary and Sufficient Conditions, Physics Letters A 223, 1-8 (1996)

1- transposition of the whole state (preserves positivity - ok!)

$$\rho = \begin{pmatrix} |a|^2 & c \\ c^* & |b|^2 \end{pmatrix} \qquad \rho^T = \begin{pmatrix} |a|^2 & c^* \\ c & |b|^2 \end{pmatrix}$$

2- partial transposition (not necessarily positive - not ok!)

$\sigma = \rho_{AB}^{T_B}$ Not necessarily a quantum state!

3- however, if the state is separable then σ is still a quantum state!

$$\rho_{AB} = \sum_i p_i \rho_{Ai} \otimes \rho_{Bi} \qquad \sigma = \sum_i p_i \rho_{Ai} \otimes \rho_{Bi}^T$$

Rule to calculate negativity:

1- partially transpose the state of a bipartite system

2- calculate the eigenvalues of the new matrix. (find the negative eig. λ)

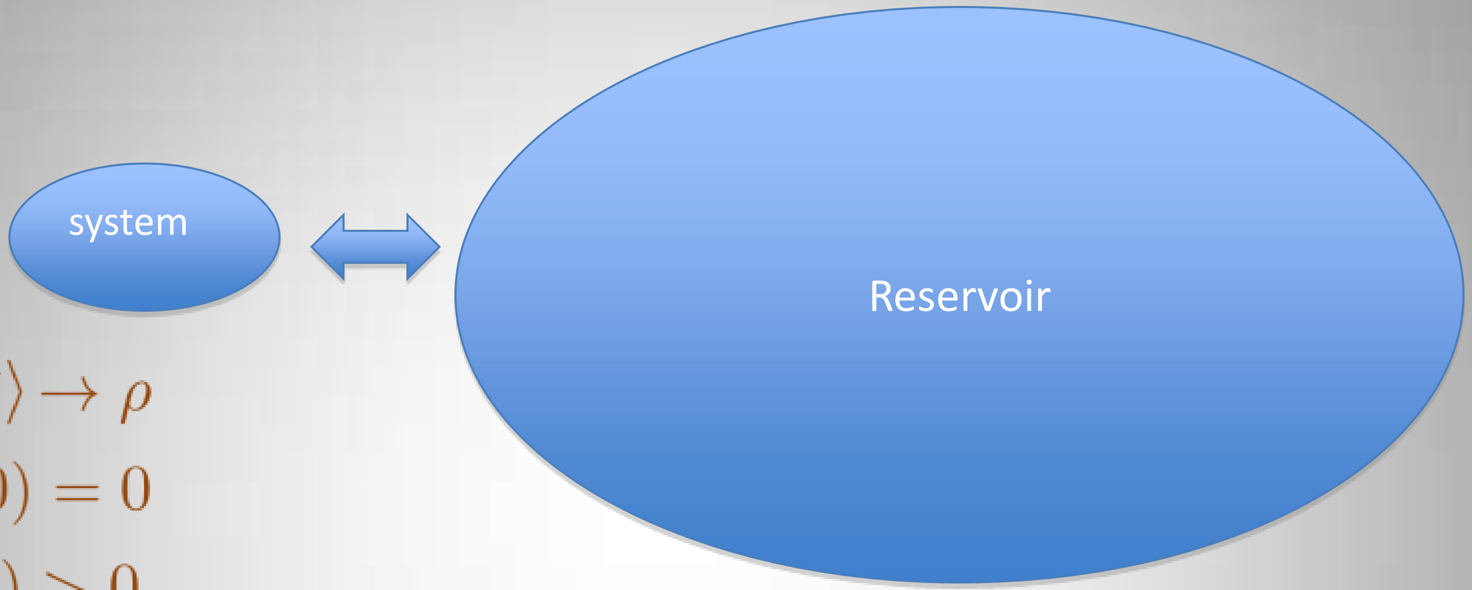
$$N = \max\{0, -2\lambda\}$$

Example:

$$|\Psi\rangle = \frac{|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle}{\sqrt{2}} \longrightarrow \rho_{AB} = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} \longrightarrow \sigma = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\lambda = -\frac{1}{2} \qquad N = 1$$

Entanglement as a spoiler: decoherence



$$|\Psi\rangle \to \rho$$

$$S(0) = 0$$

$$S(t) > 0$$

One example: spontaneous emission

$$\dot{\rho} = \frac{\gamma}{2}(2\hat{c}\rho\hat{c}^\dagger - \hat{c}^\dagger\hat{c}\rho - \rho\hat{c}^\dagger\hat{c})$$

$$\hat{c} \equiv |0_s\rangle\langle 1_s|$$

Equivalent map (one possible purification):

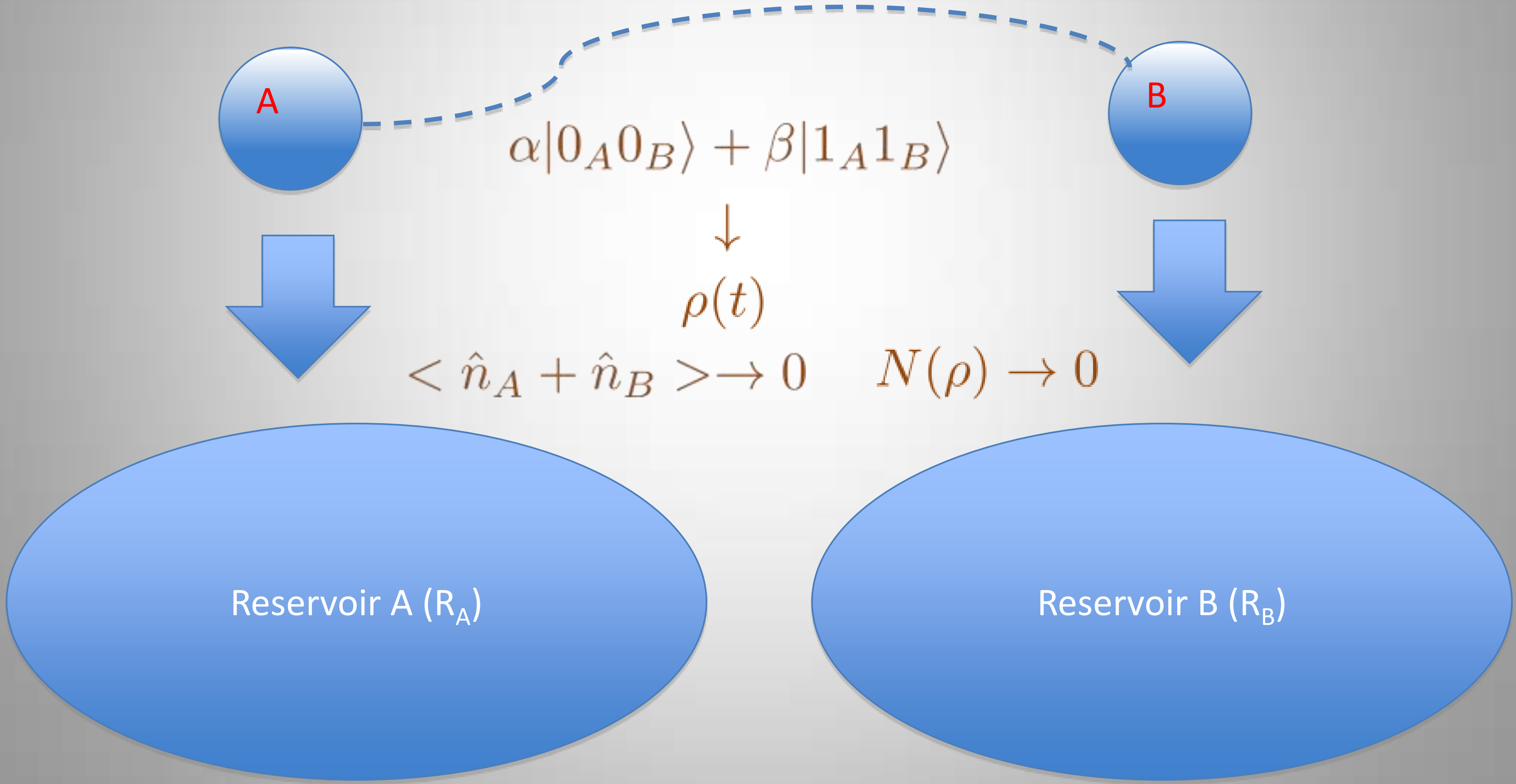
$$|1_s\rangle|0_R\rangle \to \sqrt{1-p}|1_s\rangle|0_R\rangle + \sqrt{p}|0_s\rangle|1_R\rangle$$

$$|0_s\rangle|0_R\rangle \to |0_s\rangle|0_R\rangle$$

$$p = 1 - e^{-\gamma t}$$

Disentanglement

Two qubits losing energy and entanglement to independent reservoirs



Observing disentanglement (a do it yourself recipe)

- Prepare state: $|\Psi\rangle = \alpha|00\rangle + \beta|11\rangle$

- Photon polarization
- Trapped ions
- NMR
- Microwave and optical cavity QED
- Cold atoms
- Quantum dots
- Solid state coupled cavities
- etc

M. P. Almeida, F. de Melo, M. Hor-Meyll, A. Salles, S. P. Walborn, P. H. Souto Ribeiro, and L. Davidovich, *Environment-Induced Sudden Death of Entanglement, Science, vol. 316, , 579 (2007)*

Evolution of the two-qubit system

- Density matrix at time *t* :

$|11\rangle\langle 11|$ $|10\rangle\langle 10|$

$$\rho(t) = \begin{pmatrix} w(t) & 0 & 0 & z(t) \\ 0 & x(t) & 0 & 0 \\ 0 & 0 & x(t) & 0 \\ z(t) & 0 & 0 & y(t) \end{pmatrix} \quad \sigma(t) = \begin{pmatrix} w(t) & 0 & 0 & 0 \\ 0 & x(t) & z(t) & 0 \\ 0 & z(t) & x(t) & 0 \\ 0 & 0 & 0 & y(t) \end{pmatrix}$$

$|00\rangle\langle 11|$ $|01\rangle\langle 01|$ $|00\rangle\langle 00|$

Initial state: $x = 0$, $w = \beta^2$, $y = \alpha^2$, $z = \alpha\beta$

$$|\Psi\rangle = \alpha|00\rangle + \beta|11\rangle$$

- Eigenvalues of σ(t): w, y, x+|z|, x-|z|
- Possible negative eigenvalue of σ(t) $\lambda(t) = x(t) - |z(t)|$
- Entanglement as long as x(t) < |z(t)| $N = 2(|z(t)| - x(t))$

Measurement of negativity (particular case)

- Let $|\Phi\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$
- Then

$$P(t) \equiv \langle\Phi|\rho_{AB}|\Phi\rangle = \frac{1}{2}[1 - 2x(t) + 2z(t)] = \frac{1}{2}[1 - 2\lambda(t)]$$

$$\rho(t) = \begin{pmatrix} w(t) & 0 & 0 & z(t) \\ 0 & x(t) & 0 & 0 \\ 0 & 0 & x(t) & 0 \\ z(t) & 0 & 0 & y(t) \end{pmatrix}$$

$$N = \max\{0, -2\lambda\}$$

$$N(t) = \max\{0, 2P(t) - 1\}$$

↓

Disentanglement when $P(t)$ reaches ½!

Measurement of negativity (particular case)

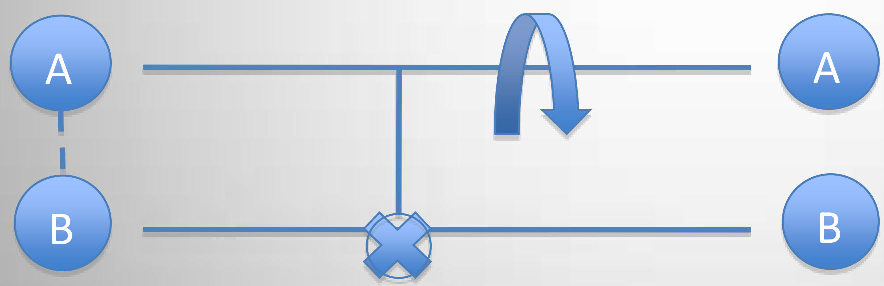
- And how to measure the projection on $|\Phi\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$

- Control-not gate:

$$
\begin{aligned}
|00\rangle &\to |00\rangle \\
|01\rangle &\to |01\rangle \\
|10\rangle &\to |11\rangle \\
|11\rangle &\to |10\rangle
\end{aligned}
$$

- Haddamard gate:

$$
\begin{aligned}
|0\rangle + |1\rangle &\to |0\rangle \\
|0\rangle - |1\rangle &\to |1\rangle
\end{aligned}
$$



$$
\begin{aligned}
|00\rangle + |11\rangle &\to (|0\rangle + |1\rangle)|0\rangle \to |00\rangle \\
|00\rangle - |11\rangle &\to (|0\rangle - |1\rangle)|0\rangle \to |10\rangle \\
|01\rangle + |10\rangle &\to (|0\rangle + |1\rangle)|1\rangle \to |01\rangle \\
|01\rangle - |10\rangle &\to (|0\rangle - |1\rangle)|1\rangle \to |11\rangle
\end{aligned}
$$

Measure the ground state population of the individual qubits!

Analytic solution

Initial state: $x = 0$, $w = \beta^2$, $y = \alpha^2$, $z = \alpha\beta$

$$\rho(t) = \begin{pmatrix} w(t) & 0 & 0 & z(t) \\ 0 & x(t) & 0 & 0 \\ 0 & 0 & x(t) & 0 \\ z(t) & 0 & 0 & y(t) \end{pmatrix}$$

$$\begin{aligned} w(t) &= w(0)e^{-\gamma t} \\ x(t) &= w(0)(1-e^{-\gamma t})e^{-\gamma t} \\ y(t) &= y(0) + w(0)(1-e^{-\gamma t})^2 \\ z(t) &= z(0)e^{-\gamma t} \end{aligned}$$

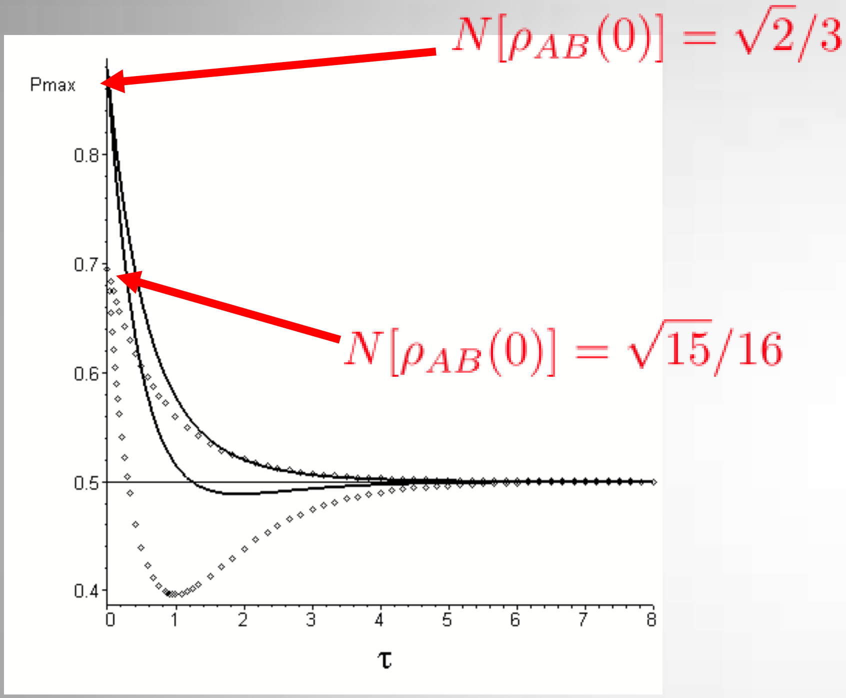
$$N = 2(|z(t)| - x(t))$$

$$x\ t = |z\ t| \Rightarrow t_s = \frac{1}{\gamma}\ln\left(1 - \frac{|\alpha|}{|\beta|}\right)$$

Separability at finite times:

$$|\beta| > |\alpha|$$

Time-dependent behavior of separability



$$|\Psi\rangle = \alpha|00\rangle + \beta|11\rangle$$

$$N(0) = 2\beta\alpha$$

$$N(t) = \max\{0, 2P(t) - 1\}$$

Finite-time disentanglement:

$$\beta > \alpha$$

MFS, P. Milman, L. Davidovich, N. Zagury, Phys. Rev. A, 73, 040305(R) (2006)

(for initially mixed states)

L. Diósi, in *Irreversible Quantum Dynamics, edited by F.Benatti and R. Floreanini (Springer, Berlin, 2003)*

P. J. Dodd and J. J. Halliwell, Phys. Rev. A ***69, 052105 (2004)***

T. Yu and J. H. Eberly, Phys. Rev. Lett. 93, 140404 (2004)

Strategies to beat decoherence due to dissipation

Redundancy and entangled ancilla:

- Quantum error correction
- Stabilizer codes
- Entanglement-Assisted Quantum Error Correction

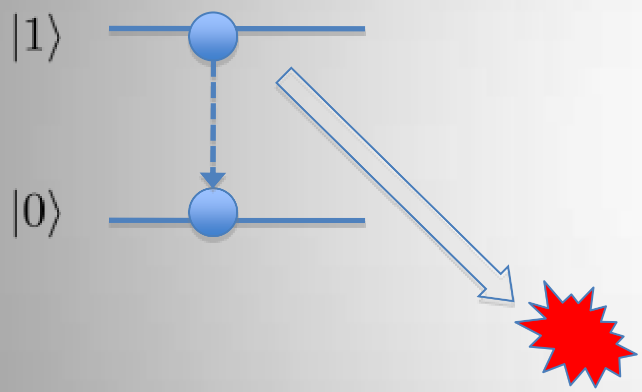
Many copies and global actions:

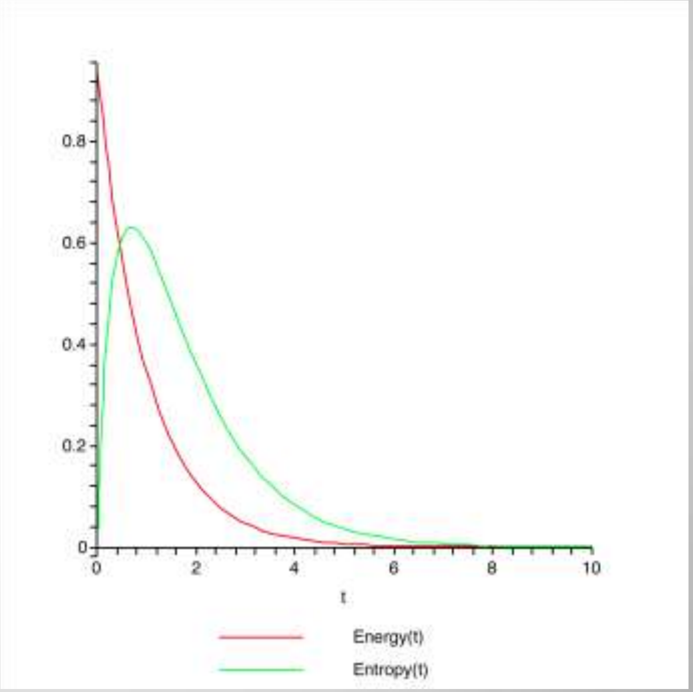
- Entanglement distillation
- Quantum repeaters
- Feedback

No local, single copy, no ancilla strategy!

Monitoring the reservoir (quantum trajectories) = acquiring information

General non-monitored time evolution:





$$|\Psi(0)\rangle = \alpha|0\rangle + \sqrt{1-\alpha^2}|1\rangle$$

$$\alpha = \frac{1}{4}$$

$$\dot{\rho} = \frac{\gamma}{2}(2\hat{c}\rho\hat{c}^\dagger - \hat{c}^\dagger\hat{c}\rho - \rho\hat{c}^\dagger\hat{c})$$

$$\hat{c} \equiv |0_s\rangle\langle 1_s|$$

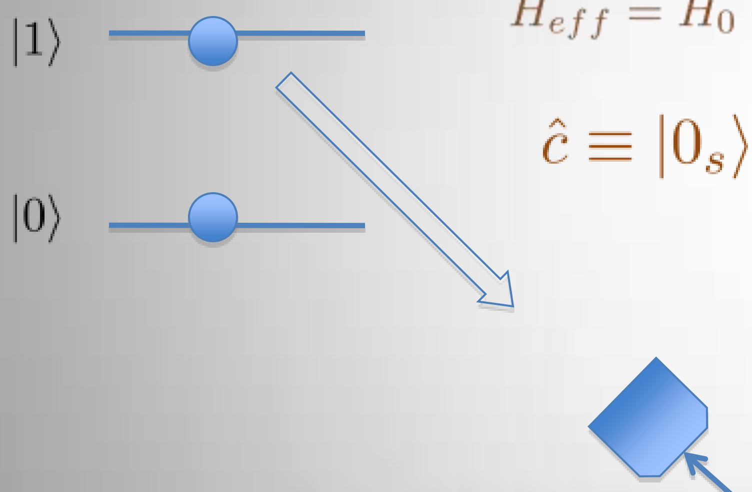
Monitoring the reservoir (quantum trajectories) = acquiring information

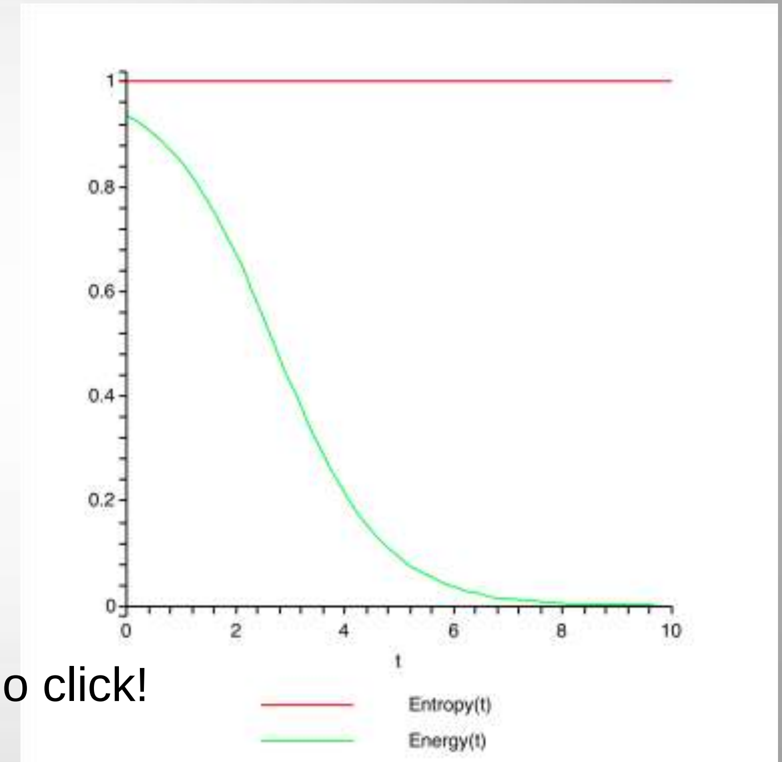
What if we look for the lost photon? (and do not find it!)

"No-jump" trajectory

$$H_{eff} = H_0 - i\frac{\gamma}{2}\hat{c}^\dagger \hat{c}$$

$$\hat{c} \equiv |0_s\rangle\langle 1_s|$$





Detector – no click!

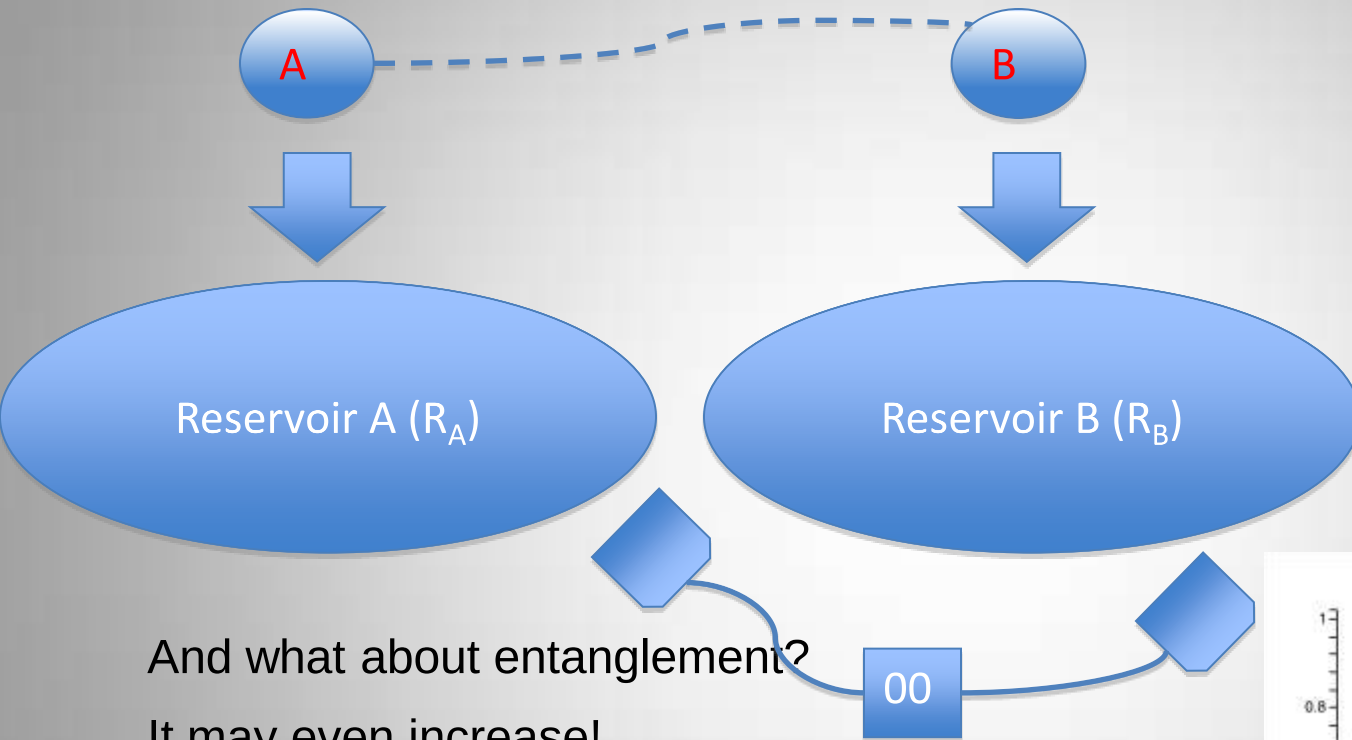
$$|\Psi(0)\rangle = \alpha|0\rangle + \sqrt{1-\alpha^2}|1\rangle$$

$$\alpha = \frac{1}{4}$$

On the other hand, one click… $|\Psi\rangle \rightarrow |0\rangle$

The same can be applied to entanglement loss!

Remember “Two qubits losing energy and entanglement”?



$$\alpha|0_A0_B\rangle + \beta|1_A1_B\rangle$$
$$\downarrow$$
$$|\Psi(t)\rangle$$
$$< \hat{n}_A + \hat{n}_B > \rightarrow 0$$

And what about entanglement?

It may even increase!

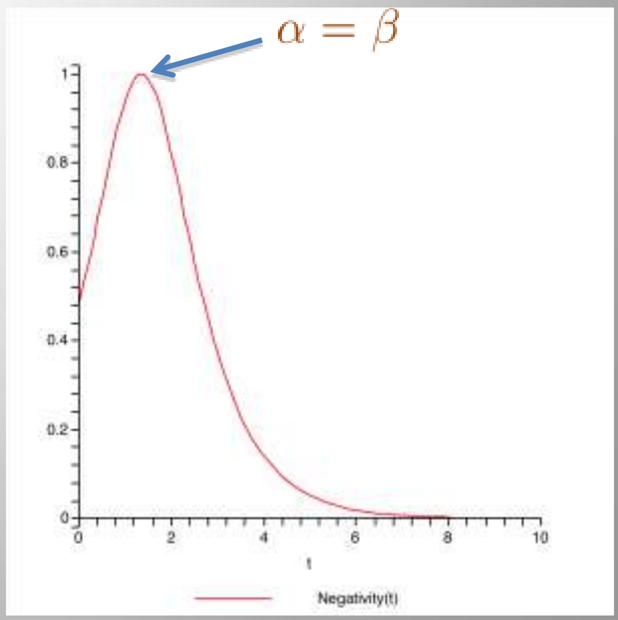
But no worries, this is equivalent to post-selection!

In other words, probabilistic! $P \propto e^{-\gamma t}$

E. Mascarenhas, B. Marques, D. Cavalcanti, M. Terra Cunha, MFS, Phys. Rev. A, 81, 032310 (2010)

Optimal singlet conversion: G. Vidal, Phys. Rev. Lett. 83, 1046 (1999).

P. G. Kwiat, S. Lopez, A. Stefanov, and N. Gisin, "Experimental entanglement distillation and 'hidden' nonlocality", Nature 409, 1014 (2001).



Once again, one click…

$$\begin{cases} |\Psi\rangle \rightarrow |01\rangle & \text{For click in } R_A \\ |\Psi\rangle \rightarrow |10\rangle & \text{For click in } R_B \end{cases}$$

What about state? $|\Psi\rangle = \alpha|10\rangle + \beta|01\rangle$

If there is no detection in either reservoirs and they dissipate in equal rates the no-jump trajectory is symmetric in this subspace

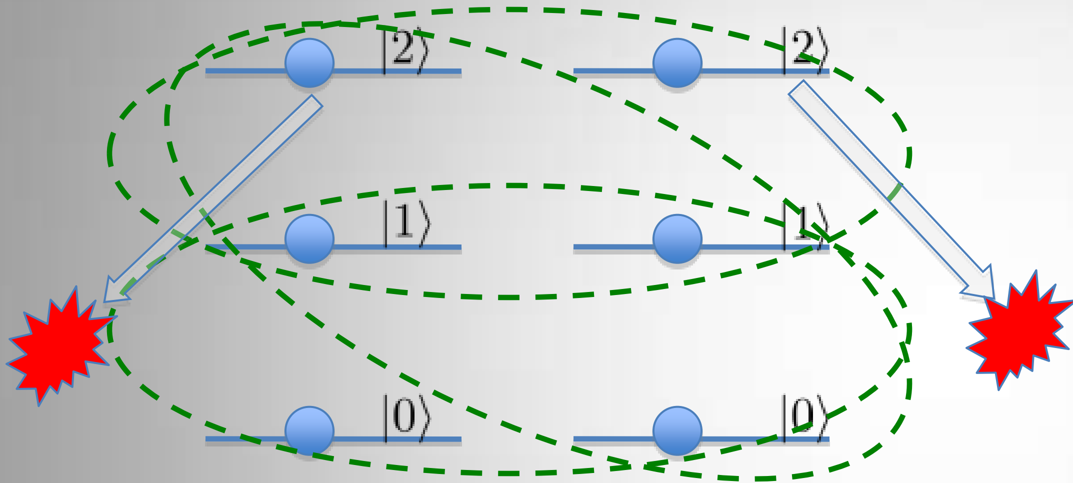
$$H_{\text{eff}} = H_0 - i\frac{\gamma}{2}\hat{c_A}^\dagger\hat{c_A} - i\frac{\gamma}{2}\hat{c_B}^\dagger\hat{c_B}$$

$$|\Psi(t)\rangle = \frac{\alpha e^{-\frac{\gamma}{2}t}|10\rangle + \beta e^{-\frac{\gamma}{2}t}|01\rangle}{\sqrt{\alpha^2 + \beta^2 e^{-\frac{\gamma}{2}t}}}$$

Entanglement is constant for the no-jump trajectory! – element # 1!

Can we do better than this?

Encode qubits in three-level systems!



$$|\Psi\rangle = \alpha|12\rangle + \beta|21\rangle$$

No detection in both reservoirs

$$\downarrow$$

$$|\Psi\rangle = \alpha|12\rangle + \beta|21\rangle$$

One jump detected in R_B:

$$\downarrow$$

$$|\Psi\rangle = \alpha|11\rangle + \beta|20\rangle$$

One jump detected in R_A:

$$\downarrow$$

$$|\Psi\rangle = \alpha|01\rangle + \beta|10\rangle$$

Requirement (there is always a catch):
-excitation detected cannot distinguish among possible decay channels:

$$\gamma_{2,1} = \gamma_{1,0} \qquad \omega_{2,1} = \omega_{1,0}$$

Example: spontaneous decay of spin 1!

And what if it does?

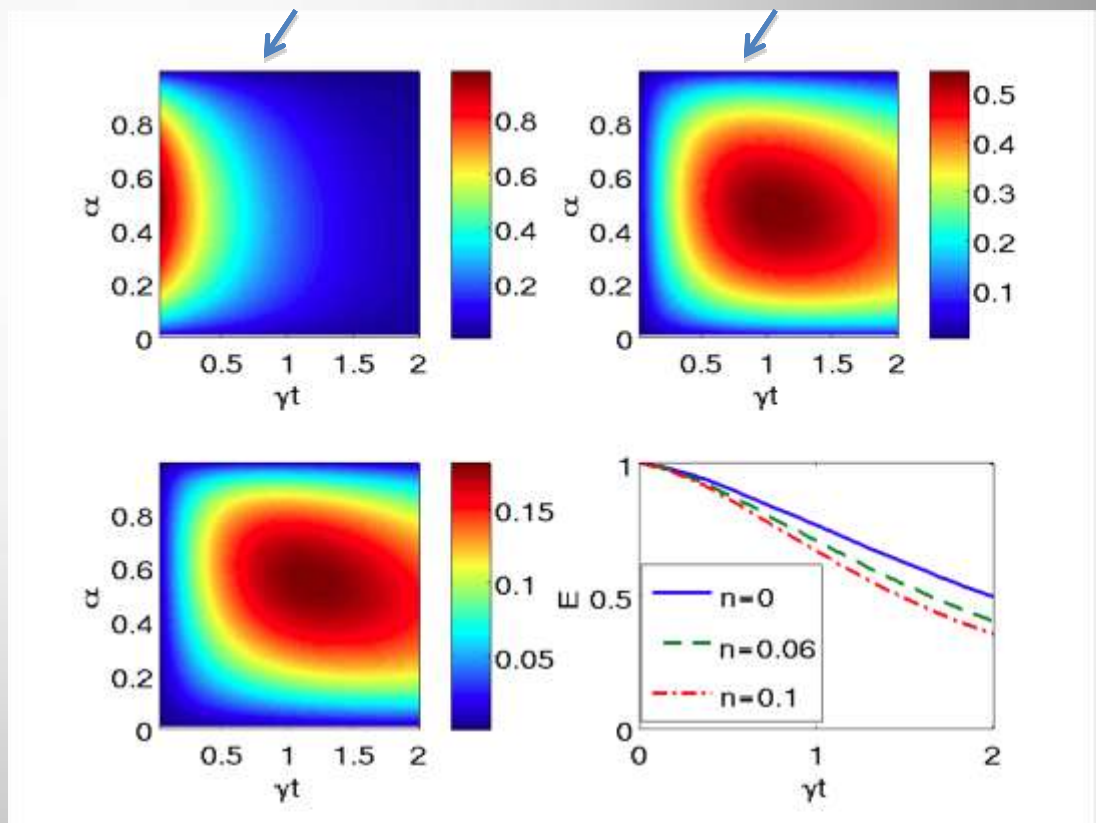
- Complete distinction destroys entanglement – projects local state into a pure one!
- Partial distinction changes entanglement. Example: harmonic oscillator.

$$H_{\text{eff}} = H_0 - i\frac{\gamma}{2}\hat{c}^\dagger\hat{c} = H_0 - i\frac{\gamma}{2}\hat{n} \qquad \gamma_{2,1} = 2\gamma_{1,0}$$

-But it's still better than two qubits!
Element # 2!

two qubits!

with qutrit!



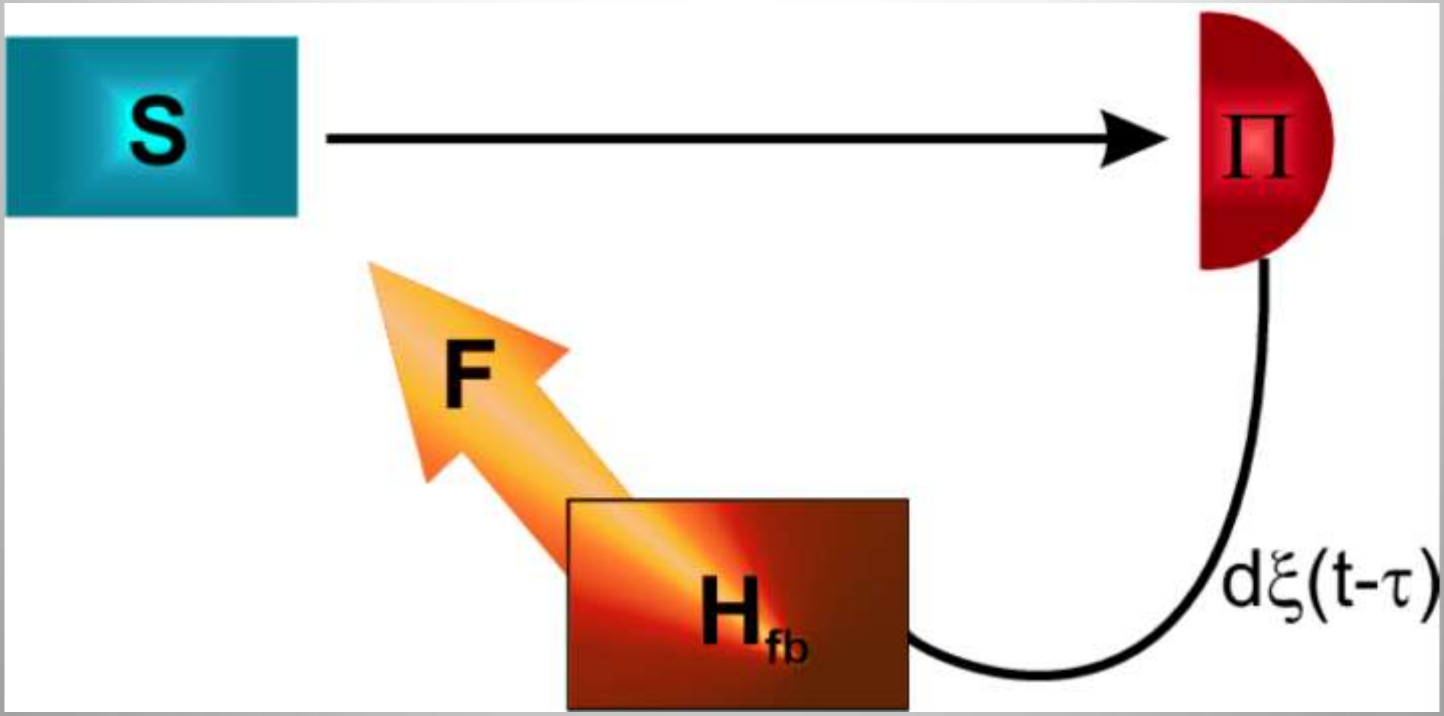
-Longer distance communication or teleportation!

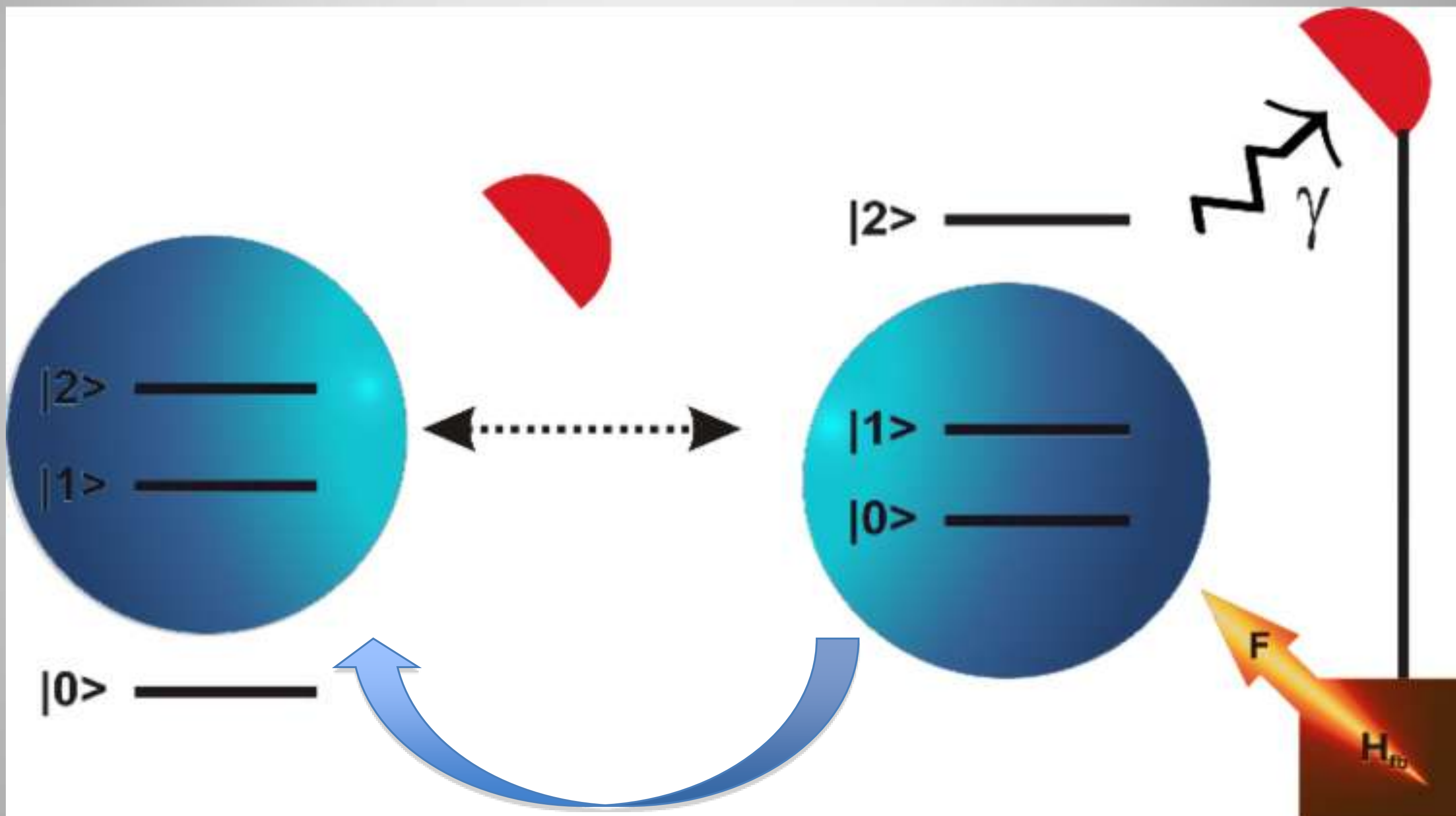
- Two detections in the same reservoir still kills entanglement!

- possibility? Keep increasing the system?!

FEEDBACK!

Element # 2!

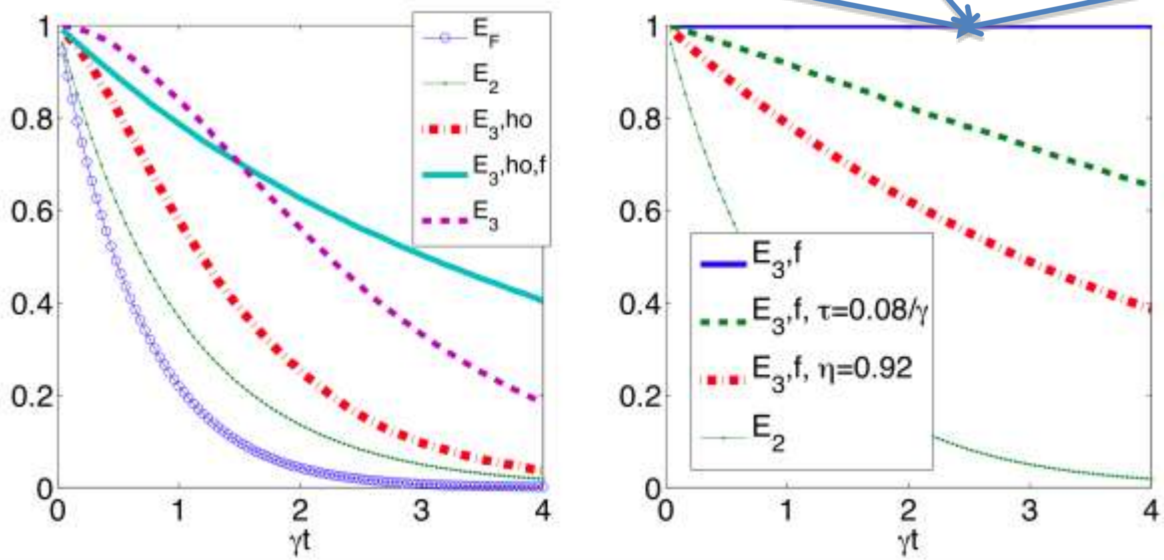



|2>
|1>
|0>
|2>
γ
|1>
|0>
F
H

Reservoir monitoring

Smart encoding

Efficient feedback



$E_{3,f}$ = 3x3, no distinction, feedback

E_3 = 3x3, no distinction, no feedback

$E_{3,ho,f}$ = 3x3, partial distinction, feedback

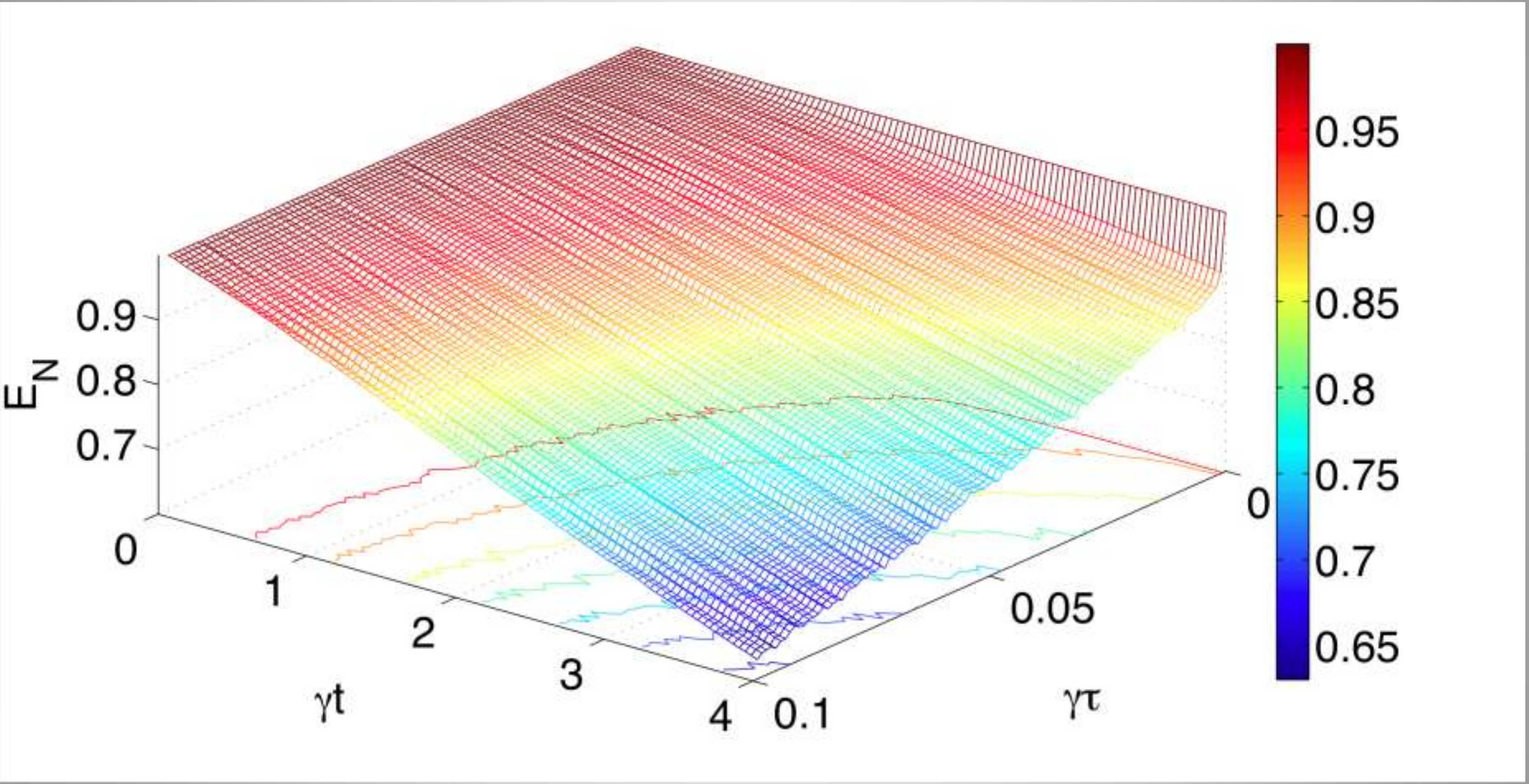
$E_{3,ho}$ = 3x3, partial distinction, no feedback

E_2 = 2x2, monitored

E_F = 2x2, not monitored

Any information is better than no information!

What if the feedback is delayed?

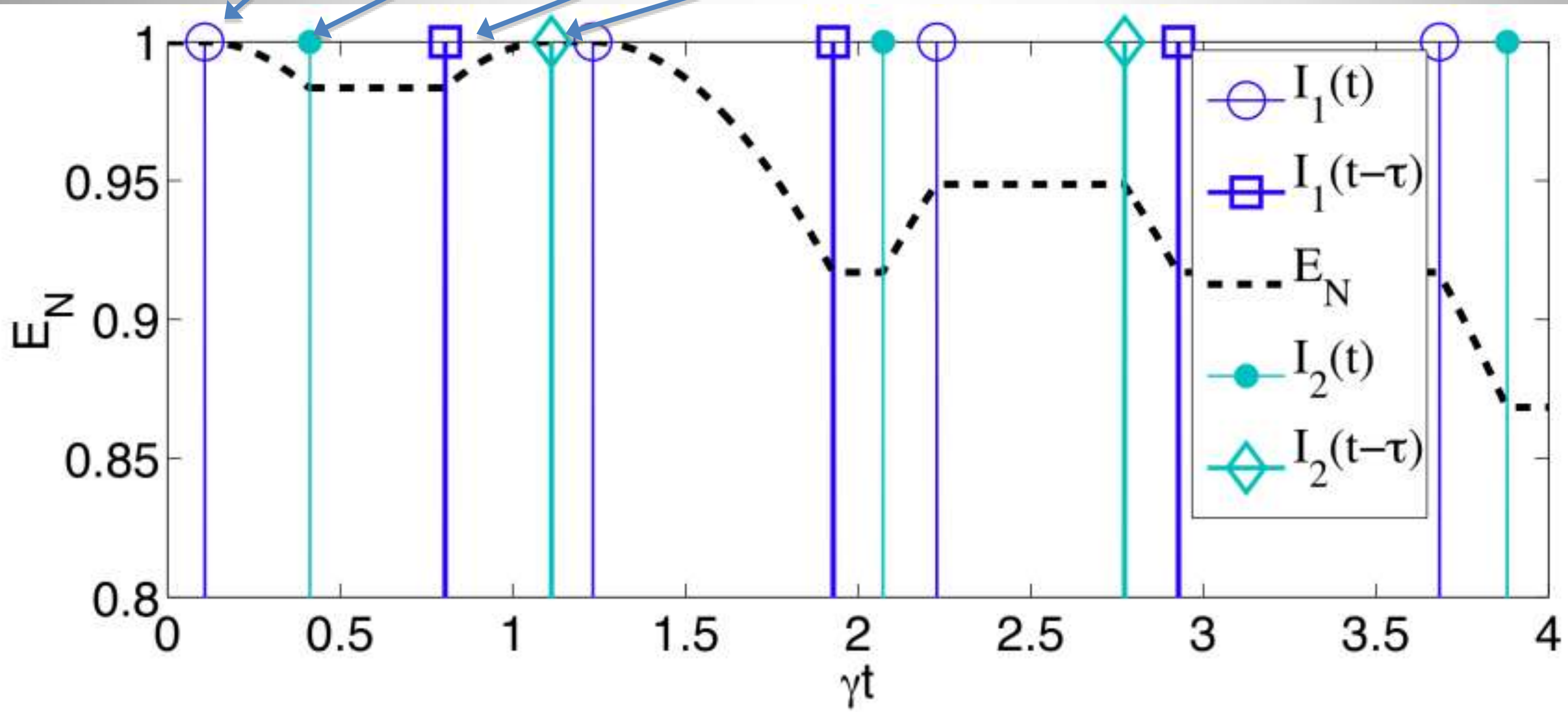


E. Mascarenhas, B. Marques, M. Terra Cunha, MFS, to be submitted

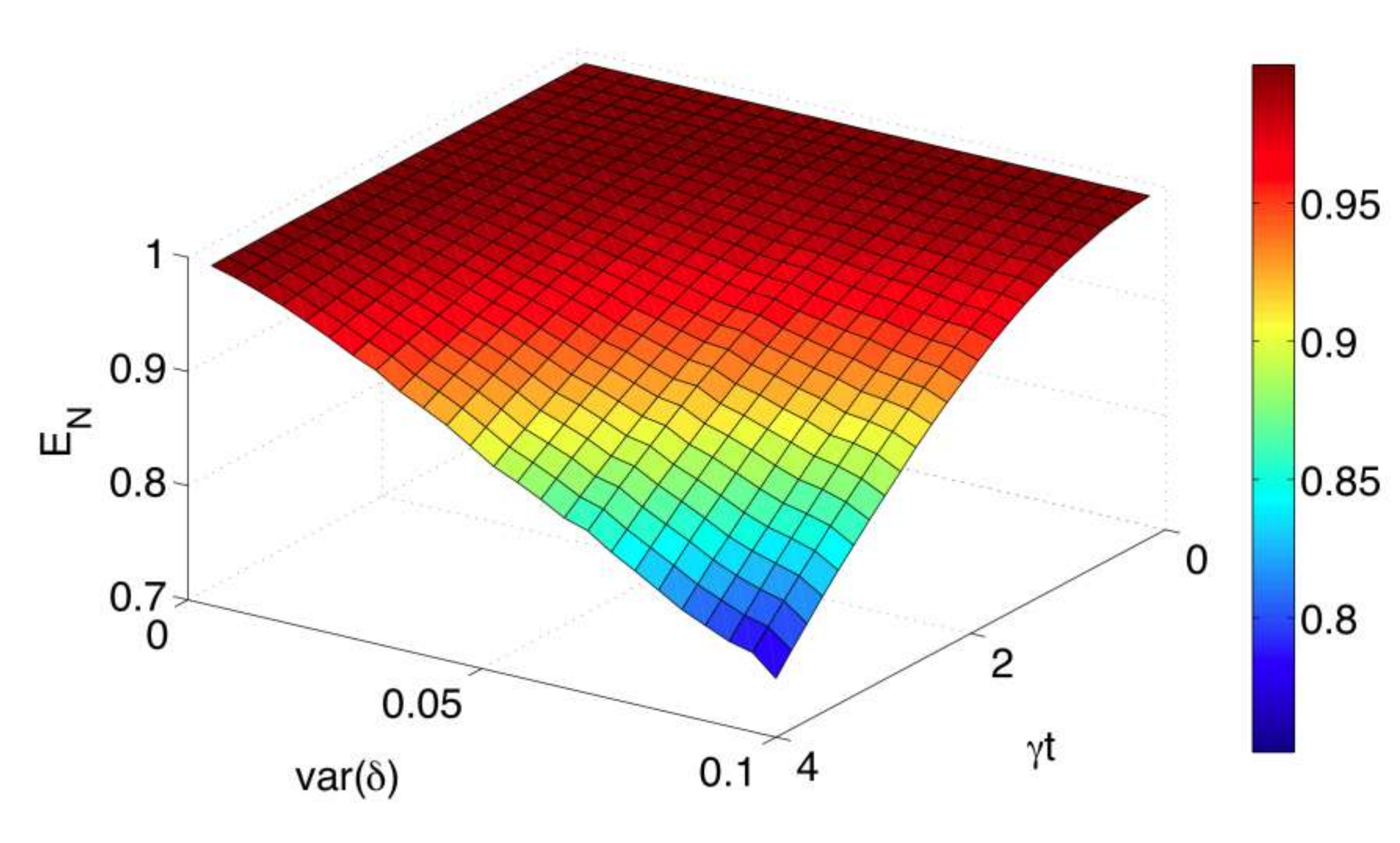
Following one trajectory (partially distinguishing detections!)

No jump – imbalance between coefficients

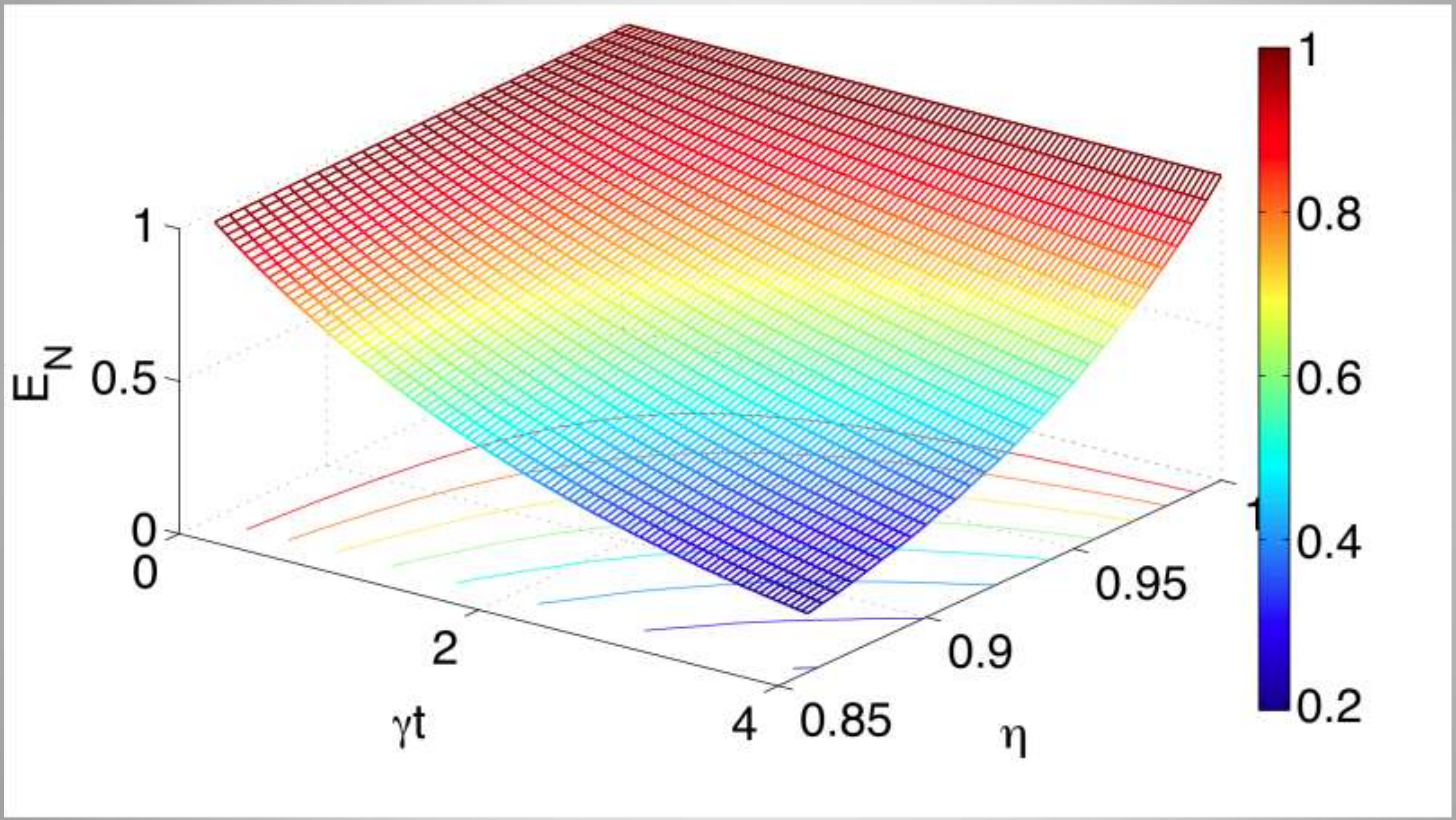
$$|\Psi_0\rangle = |12\rangle + |21\rangle \rightarrow a|02\rangle + b|11\rangle \rightarrow a|01\rangle + b|10\rangle \rightarrow |11\rangle + |20\rangle \rightarrow |12\rangle + |21\rangle$$



And what if the feedback is not ideal?



Finally, what if the detection is not efficient?



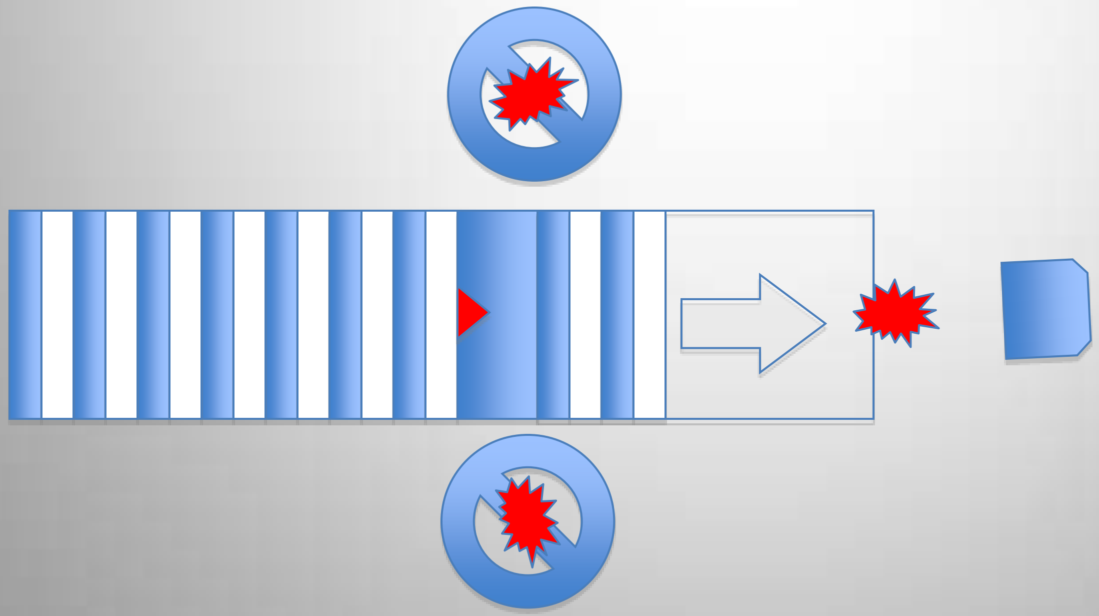
Good candidates:

Traditional candidates:

- Vibrational modes of trapped ions
- Cavity fields

More exciting one!

Excitons in quantum dots in one dimensional systems – Alexia



• Carlos Henrique Monken (exp.)
• José Geraldo de Peixoto Faria (th.)
• Marcelo de Oliveira Terra Cunha (th.)
• Marcelo França Santos (th.)
• Sebastião J. N. de Pádua (exp.)

• more than 15 graduate students
• two post-docs

• Caracterization of entanglement and its geometrical properties
• Entanglement quantifying and methods of measuring entanglement
• Decoherence and disentanglement
• Identical Particles Entanglement
• Cavity QED
• Quantum Properties of Light
• Manipulating the electromagnetic field
• Parametric Down-Conversion
• Generation of multiphotonic entangled states in birefringent media
• Properties of transverse spatial correlation of multiphotonic states and applications
• Non-classic effects of two photons states
• Quantum Information
• Open quantum systems dynamics

http://www.fisica.ufmg.br/~enlight/

Collaborators in these works:

- Luiz Davidovich (UFRJ - Brasil)
- Nicim Zagury (UFRJ - Brasil)
- Pérola Milman (CNRS – France)
- Eduardo Mascarenhas (UFMG – Brasil)
- Breno Marques (UFMG – Brasil)
- Marcelo Terra Cunha (UFMG – Brasil)
- Daniel Cavalcanti (CQT – Singapore)

- Grenoble (future works, who knows?!)

THINK GLOBAL, ACT LOCAL!

Thanks!